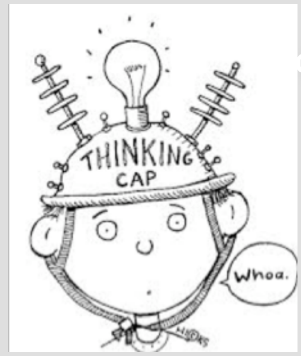
Common Core Math 3
Proofs



Can you find the error in this proof?

Given: $a = b$ **Prove:** $2 = 1$

Statement	**Reason**
1. $a = b$	Given
2. $a^2 = ab$	Multiplication property of equality (multiply both sides by *a*)
3. $a^2 - b^2 = ab - b^2$	Subtraction property of equality (subtract b^2 from both sides)
4. $(a+b)(a-b) = b(a-b)$	Substitution property of equality (substitute factored form)
5. $(a+b) = b$	Division property of equality (divide both sides by (*a-b*))
6. $a + a = a$	Substitution property of equality (substitute *a* for *b*, since *a=b*)
7. $2a = a$	Substitution property of equality (*a+a=2a*)
8. $2 = 1$	Division property of equality (divide both sides by a)

Name: ____

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

APEX HIGH SCHOOL
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Common Core Math 3

Proofs

Day	Date	Homework
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2		
3		
4		
5		
6		
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Common Core Math 3 – Proofs

Topics in this unit:

- **Algebraic properties used in geometry**
- **Relationships between angle pairs**
- **Writing a logical argument (proof) involving**
  - Lines and segments
  - Angles
  - Triangles
  - Parallel lines cut by a transversal
  - Parallelograms and rectangles
- **Geometric constructions with a compass and ruler**

By the end of this unit students will be able to:

- Use Algebraic properties when proving claims about geometric figures
- Understand relationships between angle pairs, including pairs formed by parallel lines and a transversal
- Determine values in algebraic expressions by recognizing angle relationships
- Prove theorems about lines, angles, parallelograms, and rectangles
- Use theorems about lines and angles to prove claims about geometric figures
- Create geometric constructions using a compass and ruler

VOCABULARY:

- A **postulate** is a statement that is assumed true without proof.
- A **theorem** is a true statement that can be proven.
- If point B is **between** points A and C, then A B and C are collinear.
- A **midpoint** is the point that divides a line segment into 2 congruent segments.
- A **line segment bisector is** a line, ray, or segment that cuts another line segment into two equal parts.
- **Congruent** means figures with the same size and shape.
- **Congruent segments** are two segments with equal length.
- **Congruent angles** are two angles with equal measure.
- **Congruent triangles** are triangles with corresponding sides and angles that are congruent, giving them the same size and shape
- An **angle bisector** is a line, ray, or segment that cuts an angle into two equal halves.
- P**erpendicular** segments, line, or rays that are at right angles (90°).
- **Parallel lines** lie in the same plane, and are the same distance apart over their entire length.
- A **transversal** is a line that cuts across two or more (usually parallel) lines.
- A **quadrilateral** is a 4-sided polygon.
- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.
- A **rectangle** is a parallelogram with 4 right angles.
- An **acute angle** is an angle whose measure is $< 90°$.

- **Adjacent angles** are two angles that share a vertex and a ray and no interior points.
- **Complementary angles** are two angles whose measures sum to 90°.
- A **linear pair** are two adjacent angles whose non-common sides form a line.
- An **obtuse angle** is an angle whose measure is > 90°.
- A **right angle** is an angle whose measure is = 90°.
- **Supplementary angles** are two angles whose measures sum to 180°.
- **Vertical angles** are a pair of non-adjacent angles formed by the intersection of two lines.
- **Alternate exterior angles** are two angles outside a set of parallel lines that lie on different parallel lines and are on opposite sides of a transversal
- **Alternate interior angles** are two angles inside a set of parallel lines that lie on different parallel lines and are on opposite sides of a transversal.
- **Same side exterior angles** are two angles outside a set of parallel lines that lie on different parallel lines and are the same side of a transversal.
- **Same side interior angles** are two angles inside a set of parallel lines that lie on different parallel lines and are on the same side of a transversal.
- **Corresponding angles** are two angles on the same side of the transversal in corresponding positions.

Definitions

- If B is between A and C, then A, B, and C are collinear.
- If a point divides a segment into 2 congruent segments, then it is the midpoint.
- If a point is a midpoint, then it divides the segment into two congruent segments.
- If two line segments meet or cross at right angles then they are perpendicular.
- If two line segments are perpendicular then they meet or cross at right angles.
- If two line segments have equal lengths, then they are congruent.
- If two line segments are congruent, then they have equal lengths.
- If two angles have equal measure, then they are congruent.
- If two angles are congruent, then they have equal measure.
- If a ray bisects an angle, then it divides the angle into 2 congruent angles.

Postulates (statements that are assumed true without proof)

- **Segment Addition Postulate:** If B is between A and C, then AB + BC = AC
- **Angle Addition Postulate:** If *B* is in the interior of *AOC*, then $m\angle AOB + m\angle BOC = m\angle AOC$
- **Angle Subtraction Postulate:** If *B* is in the interior of *AOC*, then $m\angle AOC - m\angle AOB = m\angle BOC$
- **Supplement Postulate**: If two angles form a linear pair then they are supplementary.
- **Side-Side-Side (SSS):** If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.
- **Side-Angle-Side (SAS):** If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
- **Angle-Side-Angle (ASA):** If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
- **Angle-Angle-Side (AAS):** If two angles and the non-included side of one triangle are congruent to the corresponding part of another triangle, the triangles are congruent.
- **Hypotenuse-Leg (HL):** If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the triangles are congruent.
- **Corresponding Angles Postulate:** If a transversal intersects two parallel lines, the pairs of corresponding angles are congruent.

Theorems (true statements that can be proven)

- If two angles are right angles, then they are congruent.
- If two angles are vertical angles, then they are congruent.
- If two angles are supplementary and congruent, then each is a right angle.
- If two angles are congruent, then their complements are congruent.
- If two angles are congruent, then their supplements are congruent.
- If two angles have the same complement (supplement), then they are congruent.
- If two angles are supplements (complements) of congruent angles, then the two angles are congruent.
- The sum of the measures of the interior angles of a triangle is 180.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
- If two sides of a triangle are congruent, the angles opposite those sides are congruent.
- If two angles of a triangle are congruent, the sides opposite those angles are congruent.
- Corresponding parts of congruent triangles are congruent (CPCTC)
- If a transversal intersects two parallel lines, then the alternate interior angles are congruent.
- If a transversal intersects two parallel lines, then the alternate exterior angles are congruent.
- If a transversal intersects two parallel lines, then same side interior angles are supplementary.
- If a transversal is perpendicular to one of 2 parallel lines, then it is perpendicular to the other one also.
- If two lines are parallel to the same line, then they are parallel to each other.
- If a quadrilateral is a parallelogram, the opposite sides are congruent.
- If a quadrilateral is a parallelogram, the opposite angles are congruent.
- If a quadrilateral is a parallelogram, the consecutive angles are supplementary.
- If a quadrilateral is a parallelogram, the diagonals bisect each other.
- If a quadrilateral is a parallelogram, the diagonals form two congruent triangles.
- If a quadrilateral is a rectangle, the diagonals are congruent.

Getting Blood From a Stone

The rules of writing a mathematical proof are very similar to the rules of "getting blood from a stone". The old saying "You can't get blood from a stone” means that nobody can give you anything that they, themselves, do not have.

The "rules of the game", and how these rules relate to mathematical proof:

1) You must start with the word STONE and end up with the word BLOOD by a series of logical steps (much like the steps in a mathematical proof).

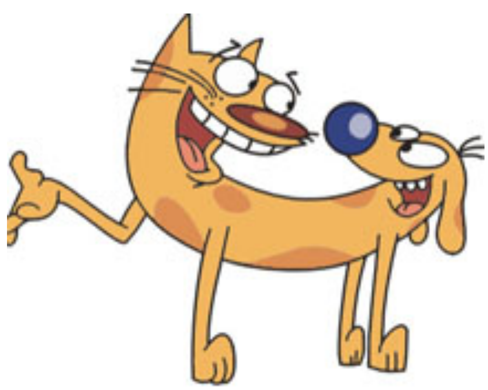
2) You must change one letter in each step. Each step must follow directly from the previous step. (In a proof, you must address one concept at a time. Each step in a proof must follow directly from the step before it - including from the 'Given', and lead to the next.)

3) The word in each step must be a real word, and in the dictionary. (In a proof, each step must be true: a definition, postulate, or theorem). If you use an unusual word, you must write the definition.

4) In this game, some people can do this in 10 steps; others may take 15, and both can be correct. There is more than one route, and no one way is necessarily better than another. One may be longer, but the best one is the one is the one that the student "sees" when trying to do the problem! (This is very true of proof; there are dozens of proofs of the Pythagorean Theorem, for example, and even in classroom assignments there is often more than one method.)

5) In this game, some people like to work backwards, or even from both ends toward the middle. (This works very well with proofs, too!)

Example:
Can you get a DOG from a CAT? (Can you go from the word CAT to the word DOG?)



Here are some possible solutions to this problem:
Solution 1: CAT - BAT - BAG - BOG - DOG
Solution 2: CAT - COT - DOT - DOG

SYMBOLS

Match the symbol in the 1^{st} column with the correct definition in the 3rd column.

Symbol	Matching Definition	Definitions
$=$		**A)** Absolute Value – it is always equal to the positive value of the number inside the lines. It represents the distance from zero.
$m\angle C$		**B)** Congruent – figures with the same size and shape.
GH		**C)** Parallel – used between segments, lines, or rays to indicate that they are always the same distance apart.
ΔABC		**D)** Line segment with endpoints G and H - line segments can be congruent to each other (you would never say they are equal).
$\perp$		**E)** Ray GH - the letter on the left indicates the endpoint of the ray.
$\angle ABC$		**F)** Equal – having the same value as another.
$\overleftrightarrow{GH}$		**G)** Plus or minus – indicates 2 values, the positive value and the negative value.
$\cong$		**H)** Triangle ABC.
$\sim$		**J)** The measure of angle C – it would equal a number.
$\overline{GH}$		**K)** Perpendicular – used between segments, line, or rays to indicate that they are at right angles (90°).
$\overrightarrow{GH}$		**L)** Angle ABC – the middle letter is always the vertex of the angle
//		**M)** Similar – figures with the same shape but not necessarily the same size.
$\pm$		**N)** The length of segment GH – it would equal a number.
$\lvert x \rvert$		**O)** The infinite line GH – lines are not equal or congruent to other lines.

Algebraic PROPERTIES used in Geometry

Property	Example(s)	
Distributive	3(n + 5) = 3n + 15 -(x – 8) = -x + 8	
Reflexive	-3 + x = -3 + x $\overline{AB} \cong \overline{AB}$	ONE EQUATION a quantity is equal (congruent) to itself
Symmetric	if y = 7 then 7 = y If $\overline{AB} \cong \overline{CD}$ then $\overline{CD} \cong \overline{AB}$	TWO EQUATIONS can switch sides of the equation (or congruence)
Transitive	If 2+3=5 and 5=10-5, then 2+3=10-5 If $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CD}$ then $\overline{AB} \cong \overline{CD}$	THREE EQUATIONS if a = b and b = c then a = c (also applies to congruence)

Properties of Equality

Substitution Property of =	if 5x - 2x + 12 = 18, then 3x + 12 = 18 If 7x + y = 9 and y = -3, then 7x – 3 = 9	If *x* = *y*, then *x* may be replaced by *y* in any equation or expression.
Addition Property of =	IF 3x – 8 = 10, then 3x = 18 If AB = DE then AB + BC = DE + BC	add an equal amount to both sides of an equation
Multiplication prop. Property of =	if $\frac{x}{3} = -4, then \ \ x = -12$	multiply both sides of an equation by an equal amount
Subtraction Property of =	if x + 5 = -6, then x = -11 If AB + BC = CD + BC, then AB = CD	subtract an equal amount from both sides of an equation
Division Property of =	if 3x = 12, then x = 4 If 3AB = 6EF, then AB = 2EF	divide both sides of an equation by an equal amount

Algebraic Properties Worksheet

I. Name the property that justifies each statement.

1. If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

2. If $x + 3 = 17$, then $x = 14$

3. $xy = xy$

4. If $7x = 42$, then $x = 6$

5. If $XY - YZ = XM$, then $XY = XM + YZ$

6. $2(x + 4) = 2x + 8$

7. If $m\angle A + m\angle B = 90$, and $m\angle A = 30$, then $30 + m\angle B = 90$.

8. If $x = y + 3$ and $y + 3 = 10$, then $x = 10$.

II. Complete the reasons in each algebraic proof.

9. Prove that if $2(x - 3) = 8$, then $x = 7$.

Given: $2(x - 3) = 8$

Prove: $x = 7$

Statements	**Reasons**
a) $2(x - 3) = 8$	a)________________________________
b) $2x - 6 = 8$	b)________________________________
c) $2x = 14$	c) ________________________________
d) $x = 7$	d) ________________________________

10. Prove that if $3x - 4 = \frac{1}{2}x + 6$, then $x = 4$.

Given: $3x - 4 = \frac{1}{2}x + 6$

Prove: $x = 4$

Statements	**Reasons**
a) $3x - 4 = \frac{1}{2}x + 6$	a) ________________________________
b) $2(3x - 4) = 2(\frac{1}{2}x + 6)$	b) ________________________________
c) $6x - 8 = x + 12$	c) ________________________________
d) $5x - 8 = 12$	d) ________________________________
e) $5x = 20$	e) ________________________________
f) $x = 4$	f) ________________________________

Name the property that justifies each statement.

________________________________1. If 3x = 120, then x = 40.

________________________________2. If 12 = AB, then AB = 12.

________________________________3. If AB = BC and BC = CD, then AB = CD.

________________________________4. If y = 75 and y = m$\angle$A, then m$\angle$A = 75.

________________________________5. If 5 = 3x – 4 , then 3x – 4 = 5.

________________________________6. If $3\left(x-\frac{5}{3}\right)=1$, then 3x – 5 = 1.

________________________________7. If m$\angle$1 = 90 and m$\angle$2 = 90, then m$\angle$1 = m$\angle$2.

________________________________8. For XY, XY = XY.

________________________________9. If EF = GH and GH = JK, then EF = JK.

________________________________10. If m$\angle$1 + 30 = 90, then m$\angle$1 = 60.

Choose the number of reason in the right column that best matches each statement in the left column.

Statements	Reasons
________A. If x – 7 = 12, then x = 19	1. Distributive Property
________B. If MK = NJ and BG = NJ, then MK = BG	2. Addition Property of Equality
________C. If y = m$\angle$5 – 30 and m$\angle$5 = 90, then y = 90 – 30	3. Symmetric Property
________D. If ST = UV, then UV = ST.	4. Substitution Property
________E. If x = -3(2x – 4), then x = -6x + 12	5. Transitive Property

Name the property that justifies each statement.

____11. If AB + BC = DE + BC, then AB = DE.

____12. m$\angle$ABC = m$\angle$ABC

____13. If XY = PQ and XY = RS, then PQ = RS.

____14. If $\frac{1}{3}x=5$, then x = 15.

____15. If 2x = 9, then x = $\frac{9}{2}$.

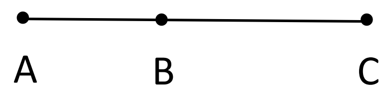
Definitions, Postulates, and Theorems

A postulate is ____.

A theorem is ____.

LINE SEGMENTS:

Between: If B is between A and C, then A, B, and C are collinear.



Line Segment Bisector: A line, ray, or segment that cuts another line segment into two equal parts.

- If a line bisects a segment, then it intersects the segment at its midpoint.

Midpoint: The point that divides a line segment into 2 congruent segments.

- If a point divides a segment into 2 congruent segments, then it is the midpoint.
- If a point is a midpoint, then it divides the segment into two congruent segments.
- If B is the midpoint of $\overline{AC}$, then AB = BC.

Perpendicular: Two segments that meet at right angles (90°)

- A line segment is perpendicular to another if it meets or crosses it at right angles.
- If two line segments meet or cross at right angles, then the segments are perpendicular.

Congruent Segments: Two segments with equal length.

- If two line segments have equal lengths, then they are congruent.
  If AB = CD then $\overline{AB} \cong \overline{CD}$
- If two line segments are congruent, then they have equal lengths.
  If $\overline{AB} \cong \overline{CD}$ then AB = CD

<u>Segment Addition Postulate</u>:

***Remember $\overline{AB}$ refers to the line segment with endpoints A and B, while AB is a numeric value, which is the length of the line segment.

☑ $\overline{AB} \cong \overline{CD}$ is correct, 2 lines segments can be congruent

☑ AB = CD is correct, the length of 2 line segments can be equal

BUT……

☒ $\overline{AB} = \overline{CD}$ is incorrect, 2 line segments are congruent, NOT equal.

☒ $AB \cong CD$ is incorrect, 2 lengths are equal, NOT congruent.

Writing Reasons

Fill in the blank with a reason for the statement. **Write your answer in "if-then" form if it is a definition.**

STATEMENTS	REASONS
1. a) $\overline{AB} \cong \overline{CD}$	a) Given
b) AB = CD	b) ____________________
2. a) M is the midpoint of $\overline{AB}$.	a) Given
b) $\overline{AM} \cong \overline{MB}$	b) ____________________
3. a) $\overleftrightarrow{XY}$ bisects $\overline{AB}$ at point X.	a) Given
b) X is the midpoint of $\overline{AB}$.	b) ____________________
4. a) $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$	a) Given
b) $\overline{AB} \cong \overline{EF}$	b) ____________________
5. a) M is between A and B, $\overline{AM} \cong \overline{MB}$	a) Given
b) M is the midpoint of $\overline{AB}$	b) ____________________
6. a) M is between C and D	a) Given
b) CM + MD = CD	b) ____________________
7. a) MN = RS	a) Given
b) $\overline{MN} \cong \overline{RS}$	b) ____________________
8. a) $\overline{AB} \cong \overline{CD}$	a) Given
b) $\overline{CD} \cong \overline{AB}$	b) ____________________
9. a) AB = CD and BC = BC	a) Given
b) AB + BC = CD + BC	b) ____________________

A proof is a logical argument that establishes the truth of a statement.

Presenting a proof:

A proof is a written account of the complete thought process that is used to reach a conclusion. Each step of the process is supported by a theorem, postulate or definition verifying why the step is possible. Proofs are developed such that each step in the argument is in proper chronological order in relation to earlier steps. No steps can be left out.

There are three classic styles for presenting proofs:

Method 1: The Paragraph Proof…is a detailed paragraph explaining the proof process. The paragraph is lengthy and contains steps and reasons that lead to the final conclusion. Be careful when using this method -- it is easy to leave out critical steps (or supporting reasons) if you are not careful.

Method 2: The Flowchart Proof … is a concept map that shows the statements and reasons needed for a proof using boxes and connecting arrows. Statements, written in the logical order, are placed in the boxes. The reason for each statement is placed under that box.

Method 3: The Two-Column Proof … consists of two columns where the first column contains a numbered chronological list of steps (*"statements"*) leading to the desired conclusion. The second column contains a list of *"reasons"* which support each step in the proof. These reasons are properties, theorems, postulates and definitions.

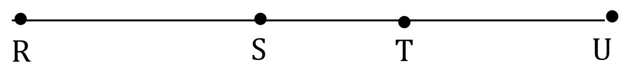
A formal 2-column proof contains the following elements:

- Statement of the original problem
- Diagram, marked with "**Given**" information
- **Re-statement** of the "Given" information in the proof
- Complete **supporting reasons** for each step in the proof
- The **"Prove"** statement as the last statement

EXAMPLE:

Given: $\overline{RT} \cong \overline{SU}$

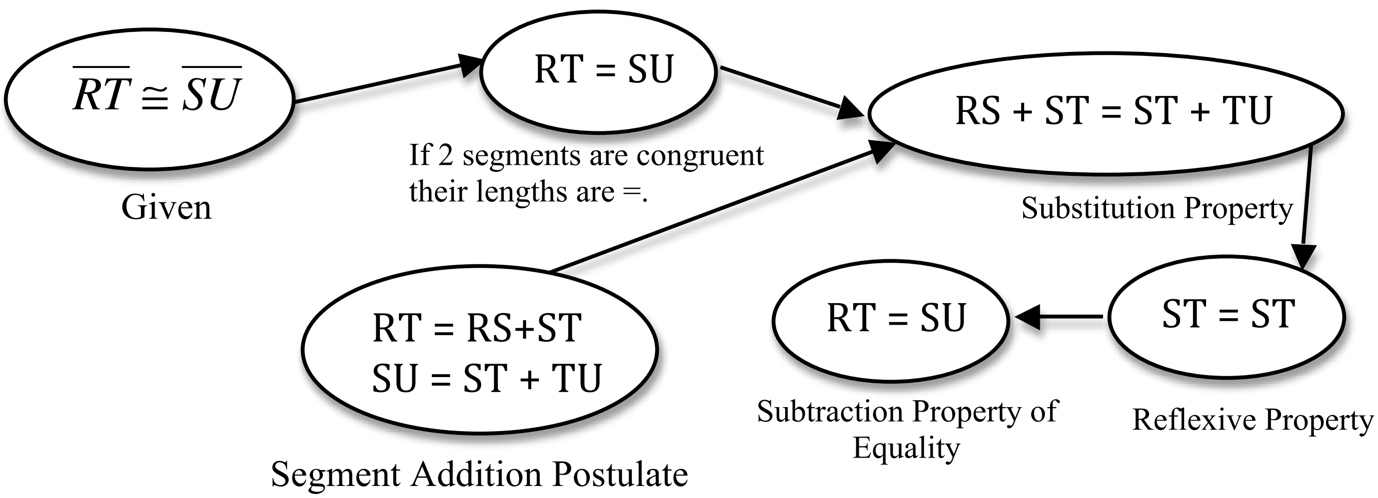
Prove: RS = TU



Method 1 - Paragraph Proof:

We are given that $\overline{RT} \cong \overline{SU}$. By definition congruent segments have equal lengths, thus RT = SU. Because S is between R and T, the Segment Addition Postulate tells us the RT = RS + ST. Because T is between S and U, the Segment Addition Postulate tells us the SU = ST + TU. We can thus substitute RS + ST for RT, and ST + TU for SU into the equality RT=SU to give RS + ST = ST + TU. ST = ST by the reflexive property. Using the subtraction property of equality, we can thus subtract ST from both sides of the equality RS + ST = ST + TU to prove that RS = TU.

Method 2 - Flowchart Proof:



Method 3 - Two-Column Proof:

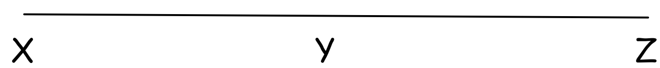
Statements	Reasons
1. $\overline{RT} \cong \overline{SU}$	Given
2. RT = SU	If two segments are congruent, then their lengths are equal.
3. RT = RS + ST	Segment Addition Postulate
4. SU = ST + TU	Segment Addition Postulate
5. RS + ST = ST + TU	Substitution Property
6. ST = ST	Reflexive Property
7. RS = TU	Subtraction property of Equality

Definition Proofs

EXAMPLE:

Given: XY = YZ

Prove: Y is the midpoint of $\overline{XZ}$



STATEMENT	REASON
1. XY = YZ	1. Given
2. $\overline{XY} \cong \overline{YZ}$	2. If 2 segments have = lengths, then they are ≅.
3. Y is the midpoint of $\overline{XZ}$	3. If a point divides a segment into 2 ≅ segments, then it is the midpoint

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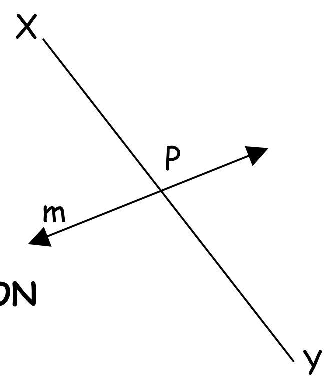
1. Given: Line m bisects $\overline{XY}$

Prove: XP = YP

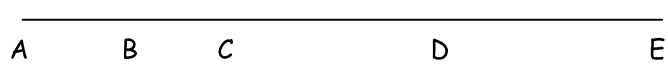


STATEMENT	REASON
1. line m bisects $\overline{XY}$.	
2. P is the midpoint of $\overline{XY}$.	
3. $\overline{XP} \cong \overline{YP}$	
4. XP = YP	

2. Given: B is the midpoint of $\overline{AC}$

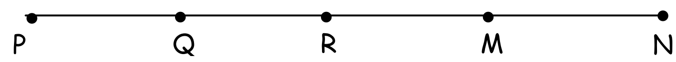
D is the midpoint of $\overline{CE}$

Prove: AB + CD = BC + DE



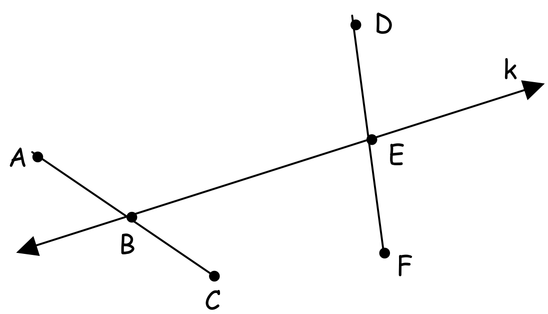
STATEMENT	REASON
1. B is the midpoint of $\overline{AC}$ D is the midpoint of $\overline{CE}$	
2. $\overline{AB} \cong \overline{BC}$ $\overline{CD} \cong \overline{DE}$	
3. AB = BC CD = DE	
4. AB + CD = BC + DE	

3. Given: PQ = MN
MN = QR
Prove: Q is the midpoint of $\overline{PR}$



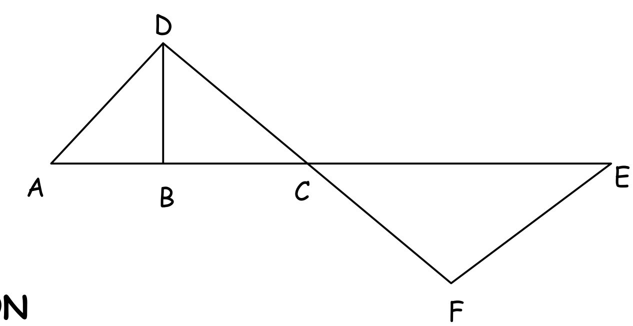
STATEMENT **REASON**

4. Given: line k bisects $\overline{AC}$
line k bisects $\overline{DF}$
Prove: AB + DE = BC + EF



STATEMENT **REASON**

5. Given: C is the midpoint of $\overline{AE}$
Prove: AB + BC = CE

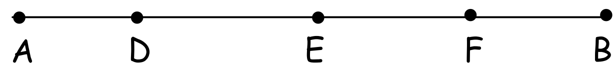


STATEMENT **REASON**

Practice with Segment Proofs

Fill in the reason for each statement below.
Then rewrite the proof as a FLOWCHART PROOF.

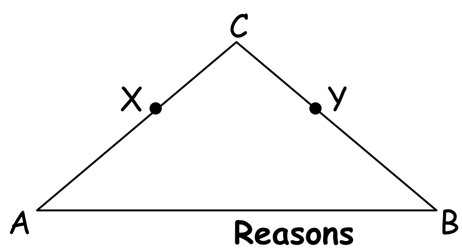
1.
Given: DE = EF, AD = BF
Prove: AE = BE



Statements	**Reasons**
1. AD = BF DE = EF	1.
2. AD + DE = BF + FE	2.
3. AD + DE = AE BE = BF + FE	3.
4. AE = BE	4.

FLOWCHART PROOF:

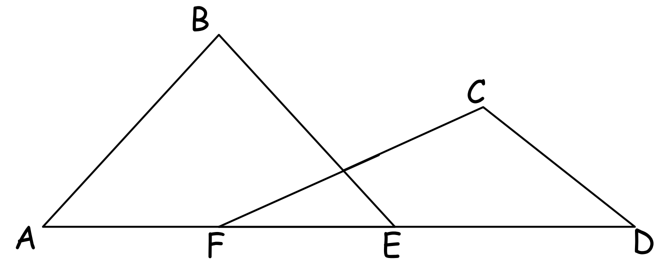
2.
Given: AX = BY, XC = YC
Prove: $\overline{AC} \cong \overline{BC}$



Statements	**Reasons**
1. AX = BY XC = YC	1.
2. AX + XC = BY + YC	2.
3. AX + XC = AC BY + YC = BC	3.
4. AC = BC	4.
5. $\overline{AC} \cong \overline{BC}$	5.

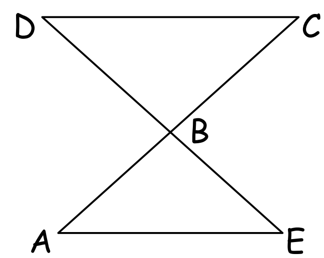
FLOWCHART PROOF:

Write 2-column proofs (from scratch)



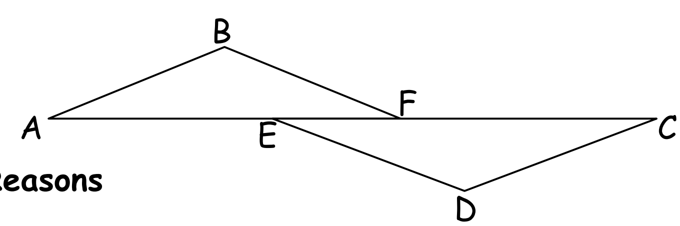
3. Given: AE = FD
   Prove: AF = ED

Statements **Reasons**



4. Given: AB = BE
   BC = BD
   Prove: AC = ED

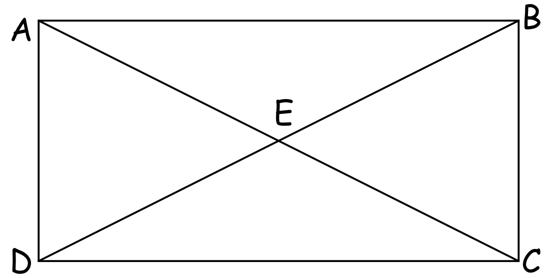
Statements **Reasons**



5. Given: AF = EC
   Prove: AE = FC

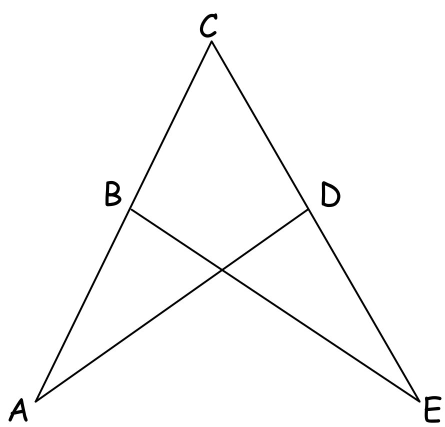
Statements **Reasons**

6. Given: AC = BD
BE = EC
Prove: AE = DE



Statements **Reasons**

7. Given: AB = DE
BC = CD
Prove: $\overline{AC} \cong \overline{EC}$



Statements **Reasons**

Angle: the union of 2 non-collinear rays with a common endpoint

Give an example or draw a diagram of each angle or angle pair.

	Definition	**Example or Diagram**
acute angle	an angle whose measure is < 90°	
adjacent angles	two angles that share a vertex and a ray and no interior points	
complementary angles	two angles are complementary if the sum of their measures is 90°	
linear pair	two adjacent angles whose non-common sides form a line	
obtuse angle	an angle whose measure is > 90°	
right angle	an angle whose measure is = 90°	
supplementary angles	two angles are supplementary if the sum of their measures is 180°	
vertical angles	a pair of non-adjacent angles formed by the intersection of two lines	

Definitions, Postulates, and Theorems

ANGLES:

Congruent Angles: Two angles with equal measure.

- If two angles have equal measure, then they are congruent.
  If $m\angle A = m\angle B$ then $\angle A \cong \angle B$
- If two angles are congruent, then they have equal measure.
  If $\angle A \cong \angle B$ then $m\angle A = m\angle B$

Angle Bisector: A line, ray, or segment which cuts an angle into two equal halves.

- If a ray bisects an angle, then it divides the angle into 2 congruent angles.

Angle Addition Postulate:

Angle Subtraction Postulate:

Supplement Postulate:

Theorems:

***Remember $\angle A$ refers to the angle, while $m\angle A$ is a numeric value, which is the measure of the angle.

☑ $\angle A \cong \angle B$ is correct, angles can be congruent.
☑ $m\angle A = m\angle B$ is correct, the measures of angles can be equal.

BUT……

☒ $\angle A = \angle B$ is incorrect, 2 angles are congruent but NOT equal.
☒ $m\angle A \cong m\angle B$ is incorrect, 2 angle measures are equal, NOT congruent.

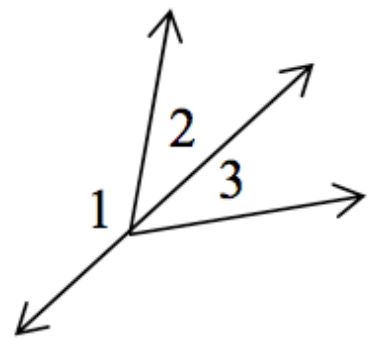
EXAMPLE:

Given:

$\angle 1$ and $\angle 2$ are supplementary
$\angle 2 \cong \angle 3$

Prove:

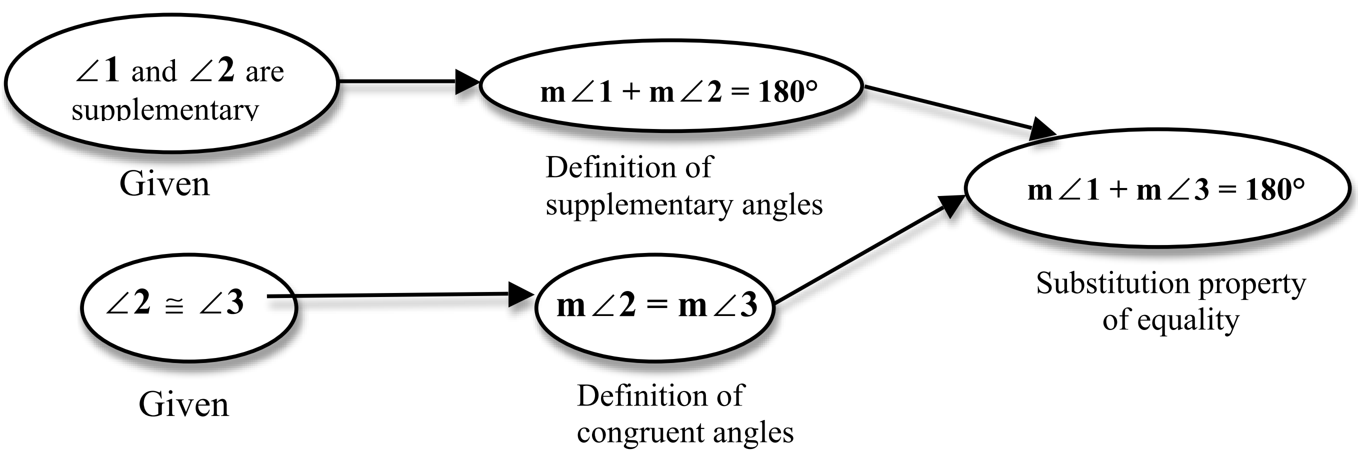
$m\angle 1 + m\angle 3 = 180°$



Method 1 - Paragraph Proof:

We are given that $\angle 1$ and $\angle 2$ are supplementary and that $\angle 2 \cong \angle 3$. By definition the measures of supplementary angles add to 180° so $m\angle 1 + m\angle 2 = 180°$. By definition congruent angles have equal measures so $m\angle 2 = m\angle 3$. We can use the substitution property of equality to substitute $m\angle 3$ for $m\angle 2$ in the equation $m\angle 1 + m\angle 2 = 180°$ and prove that $m\angle 1 + m\angle 3 = 180°$.

Method 2 - Flowchart Proof:



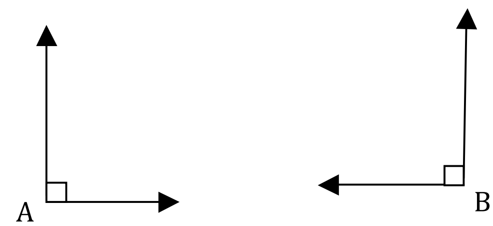
Method 3 - Two-Column Proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary	Given
2. $\angle 2 \cong \angle 3$	Given
3. $m\angle 1 + m\angle 2 = 180°$	Definition of supplementary angles
4. $m\angle 2 = m\angle 3$	If 2 angles are congruent then they have equal measure.
5. $m\angle 1 + m\angle 3 = 180°$	Substitution property of equality

Prove Angle Theorems

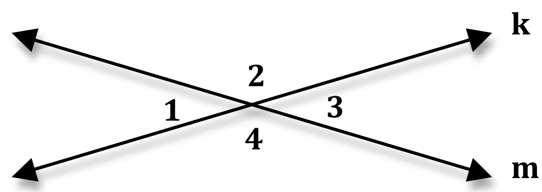
If two angles are right angles, then they are congruent. Prove this theorem.

1. Given: ∠A and ∠B are right angles
   Prove: ∠A ≅ ∠B



If two angles are vertical angles, then they are congruent. Prove this theorem.

2. Given: Lines k and m intersect
   Prove: ∠1 ≅ ∠3



If two angles have the same supplement, then they are congruent. Prove this theorem.

3. Given: ∠A and ∠B are supplementary angles
   ∠B and ∠C are supplementary angles
   Prove: ∠A ≅ ∠C

ANGLES Worksheet

ANGLE ADDITION POSTULATE: If point B lies in the interior of $\angle AOC$, then ________ + _______ = ________

If two angles form a linear pair then they are _____________________________.

All right angles are ______________.   All vertical angles are ____________________.

If two angles are ≅ and supplementary, then each is a __________ angle.

If two angles are ≅, then their ___________________ are ≅ .

If two angles are ≅, then their ___________________ are ≅

If two angles have the same complement (supplement), then they are ____________.

If two angles are supplements (complements) of ≅ angles, then the two angles are ______________.

Fill in the blank with Always, Sometimes, or Never.

__________________1. Vertical angles _________ have a common vertex.

__________________2. Two right angles are __________ complementary.

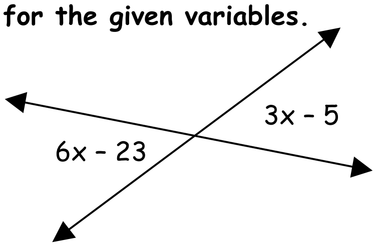
__________________3. Right angles are ___________ vertical angles.

__________________4. Angles A, B, and C are ___________ supplementary.

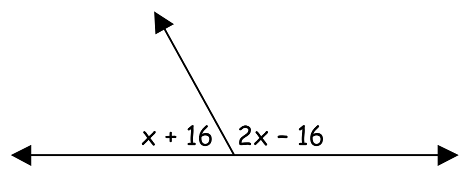
__________________5. Vertical angles ___________ have a common supplement.

Solve for the given variables.

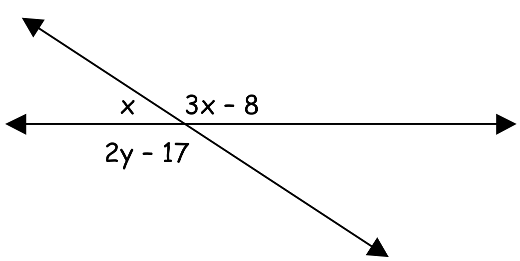
1.



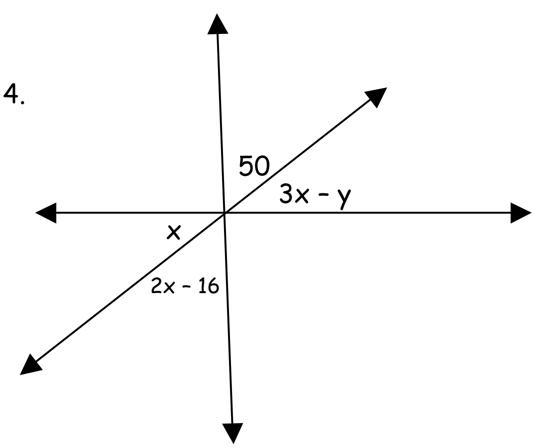
2.



3.



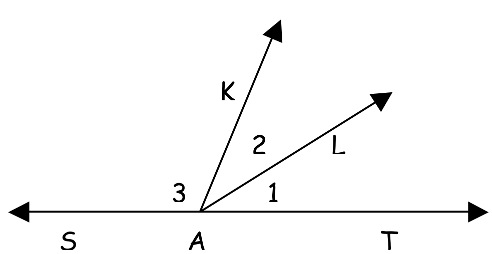
4.



5. $\overrightarrow{AL}$ **bisects** $\angle$**KAT**

$m\angle 1 = x + 12$

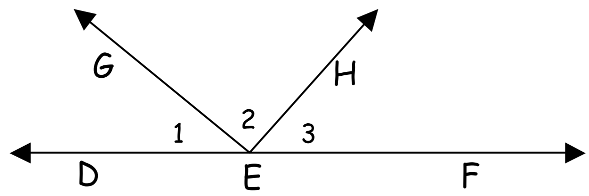
$m\angle 3 = 6x - 20$



_________________6. If $m\angle C = 40 - x$, and $\angle C$ and $\angle D$ are complementary, what is the $m\angle D$?

_________________7. If $m\angle A = x + 80$ and $\angle A$ and $\angle B$ form a linear pair, what is the $m\angle B$?

Use the figure to the right to answer #8 - 15



8. Name all angles shown.

9. Name $\angle 1$ in 2 other ways.

10. Name all angles which have side $\overrightarrow{EF}$.

11. Name a point in the interior of $\angle GEF$.

12. Name a point in the exterior of $\angle 2$.

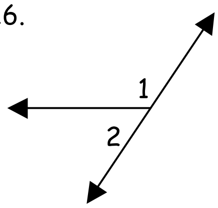
13. Name 2 pairs of adjacent angles.

14. What is the union of $\overrightarrow{ED}$ and $\overrightarrow{EG}$?

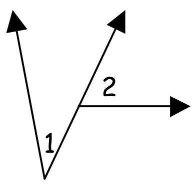
15. What is the union of $\overrightarrow{ED}$ and $\overrightarrow{EF}$?

State whether the numbered angles shown in 16 - 23 are **adjacent** or **not**. If NOT, explain.

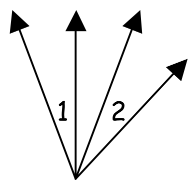
16.



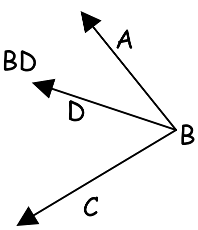
17.



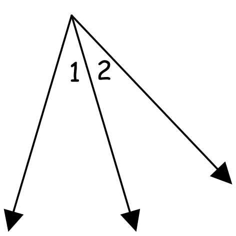
18.



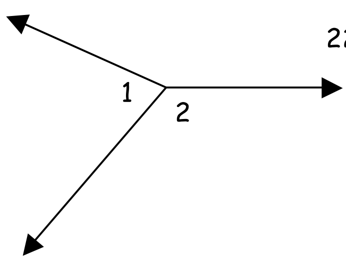
19. $\angle ABC$, $\angle ABD$



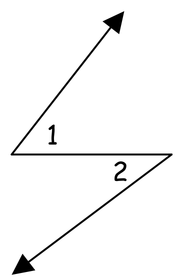
29.



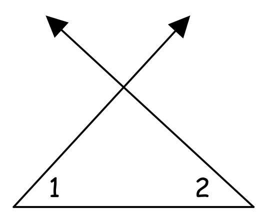
21.



22.



23.



Answers:

16. ____

17. ____

18. ____

19. ____

20. ____

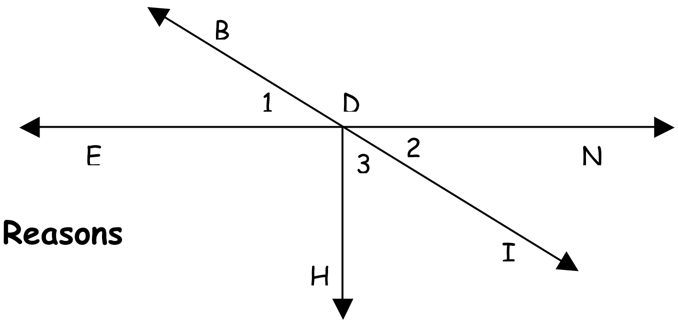
21. ____

22. ____

23. ____

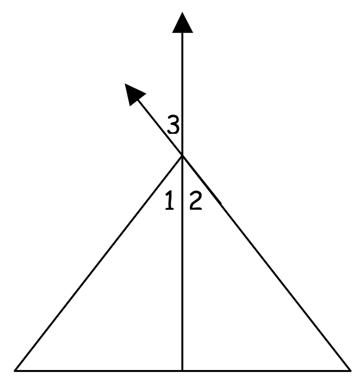
Angle Proofs Worksheet

1. Given: $\overrightarrow{DI}$ bisects $\angle NDH$
   Prove: $\angle 1 \cong \angle 3$



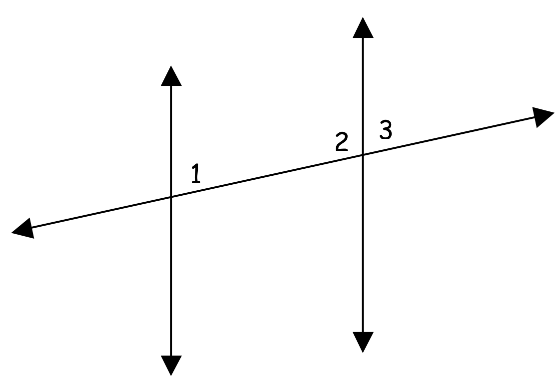
Statements **Reasons**

2. Given: $\angle 1 \cong \angle 2$
   Prove: $\angle 1 \cong \angle 3$



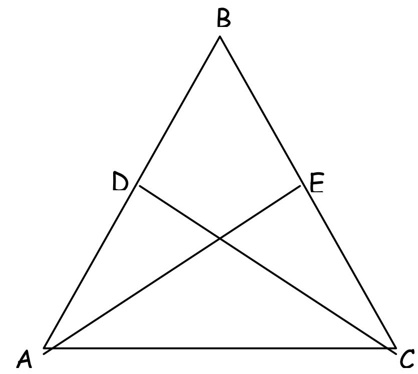
Statements **Reasons**

3. Given: $\angle 1$ is a supplement of $\angle 2$
   Prove: $\angle 1 \cong \angle 3$

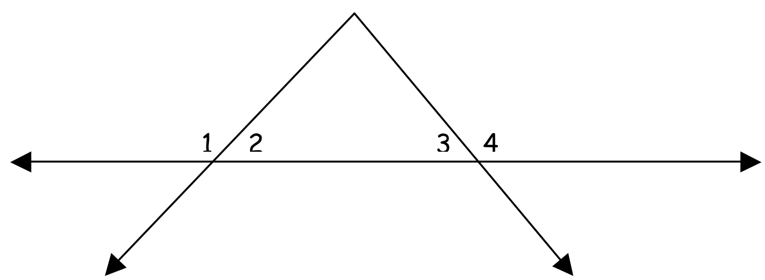


Statements	Reasons
1. $\angle 1$ *is a supplement of* $\angle 2$	1.
2. $\angle 2$ *and* $\angle 3$ *are linear pairs*	2.
3. $\angle 2$ *is a supplement of* $\angle 3$	3.
4. $\angle 1 \cong \angle 3$	4.

4. Given: ∠ADC ≅ ∠CEA
Prove: ∠CDB ≅ ∠AEB



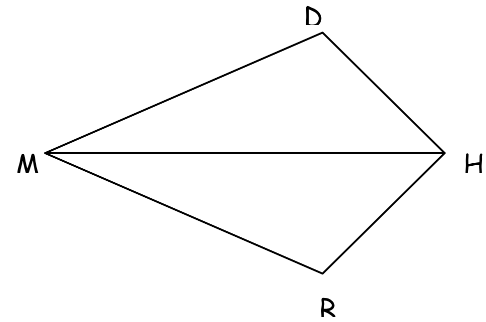
Statements **Reasons**



5. Given: ∠1 ≅ ∠4
Prove: ∠2 ≅ ∠3

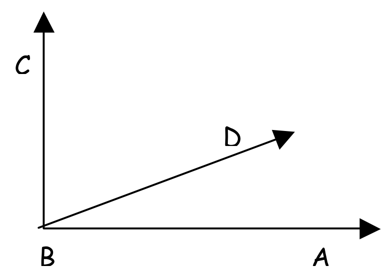
Statements **Reasons**

6. Given: $\angle DMH \cong \angle DHM$
$\angle RMH \cong \angle RHM$
Prove: $\angle DMR \cong \angle DHR$



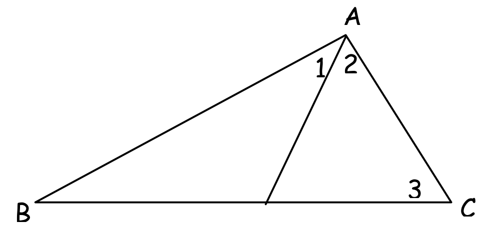
Statements	**Reasons**
1. $\angle DMH \cong \angle DHM$ $\angle RMH \cong \angle RHM$	1.
2. $m\angle DMH = m\angle DHM$ $m\angle RMH = m\angle RHM$	2.
3. $m\angle DMH + m\angle RMH = m\angle DHM + m\angle RHM$	3.
4. $m\angle DMH + m\angle RMH = m\angle DMR$ $m\angle DHM + m\angle RHM = m\angle DHR$	4.
5. $m\angle DMR = m\angle DHR$	5.
6. $\angle DMR \cong \angle DHR$	6.

7. Given: $\overline{AB} \perp \overline{BC}$
Prove: $\angle ABD$ and $\angle DBC$ are complementary



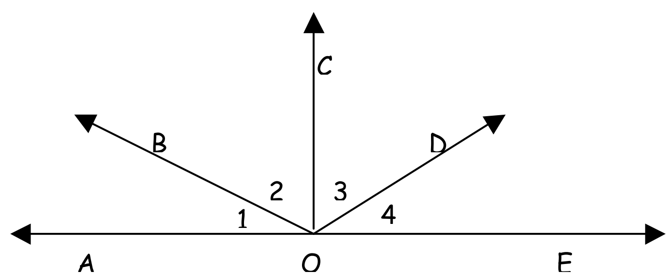
Statements	**Reasons**

8. Given: $\overline{BA} \perp \overline{CA}$; $\angle 1$ is a complement of $\angle 3$
Prove: $\angle 2 \cong \angle 3$



Statements	**Reasons**
1. $\overline{BA} \perp \overline{CA}$ $\angle 1$ is the complement of $\angle 3$	1.
2. $m\angle BAC = 90°$	2.
3. $m\angle 1 + m\angle 2 = m\angle BAC$	3.
4. $m\angle 1 + m\angle 2 = 90°$	4.
5. $m\angle 1 + m\angle 3 = 90°$	5.
6. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	6.
7. $m\angle 2 = m\angle 3$	7.
8. $\angle 2 \cong \angle 3$	8.

9. Given: $\overline{CO} \perp \overline{AE}$; $\angle 2 \cong \angle 3$
Prove: $\angle 1 \cong \angle 4$

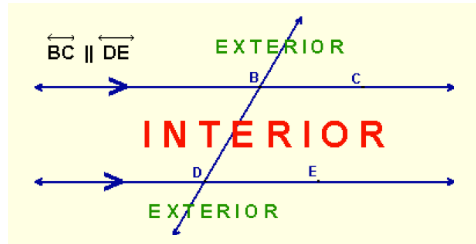


Statements	**Reasons**

Angles & Parallel Lines

A **transversal** is a line that intersects two or more lines (in the same plane). When lines intersect, angles are formed in several locations. Certain angles are given "names" that describe "where" the angles are located in relation to the lines. These names describe angles whether the lines involved are parallel or not parallel.

- the word **INTERIOR** means **BETWEEN** the lines.
- the word **EXTERIOR** means **OUTSIDE** the lines.
- the word **ALTERNATE** means **"alternating sides"** of the transversal.
- the word **CORRESPONDING** does not clearly indicate the location (They lie on the same side of the transversal, in corresponding positions.)
  - on the SAME SIDE of the transversal
  - one INTERIOR and one EXTERIOR
  - and they are NOT adjacent (*they don't touch*).



Draw a diagram of each angle pair.

	Definition	**Diagram**
alternate exterior angles	two angles outside a set of parallel lines that lie on different parallel lines and are on opposite sides of a transversal	
alternate interior angles	two angles inside a set of parallel lines that lie on different parallel lines and are on opposite sides of a transversal	
same side exterior angles	two angles outside a set of parallel lines that lie on different parallel lines and are the same side of a transversal	
same side interior angles	two angles inside a set of parallel lines that lie on different parallel lines and are on the same side of a transversal	
corresponding angles	two angles on the same side of the transversal in corresponding positions	

Definitions, Postulates, and Theorems

PARALLEL LINES:

Parallel Lines: Lines are parallel if they lie in the same plane, and are the same distance apart over their entire length. No matter how far you extend them, they will never meet.

Transversal: A line that cuts across two or more (usually parallel) lines.

Corresponding Angles Postulate:

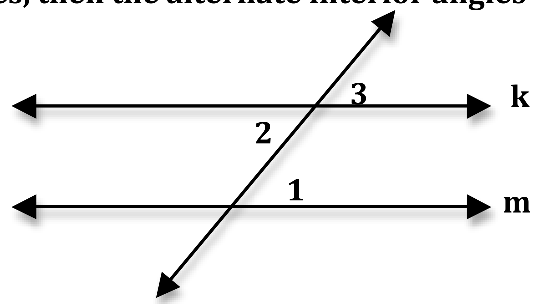
Theorems:

Prove Parallel Line Theorems

If a transversal intersects two parallel lines, then the alternate interior angles are congruent. Prove this theorem.

1. Given: k // m

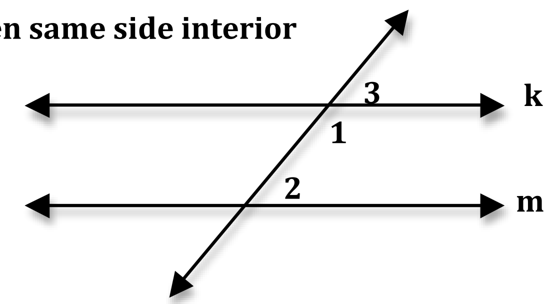
   Prove: $\angle 1 \cong \angle 2$



If a transversal intersects two parallel lines, then same side interior angles are supplementary. Prove this theorem.

2. Given: k // m

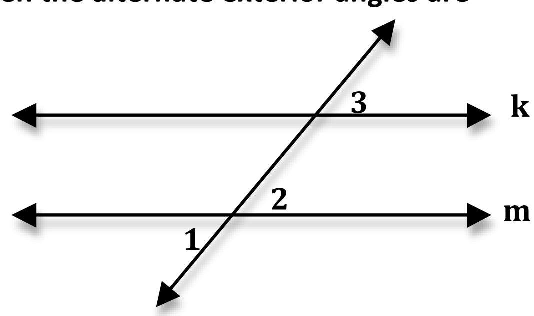
   Prove: $\angle 1 \ and \ \angle 2$ are supplementary

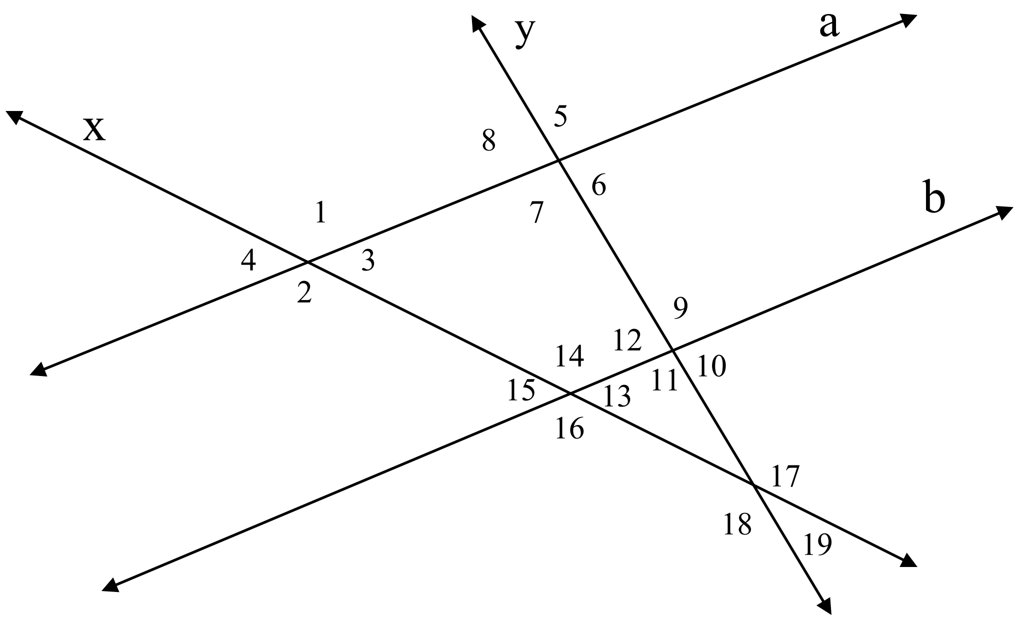


If a transversal intersects two parallel lines, then the alternate exterior angles are congruent. Prove this theorem.

3. Given: k // m

   Prove: $\angle 1 \cong \angle 3$



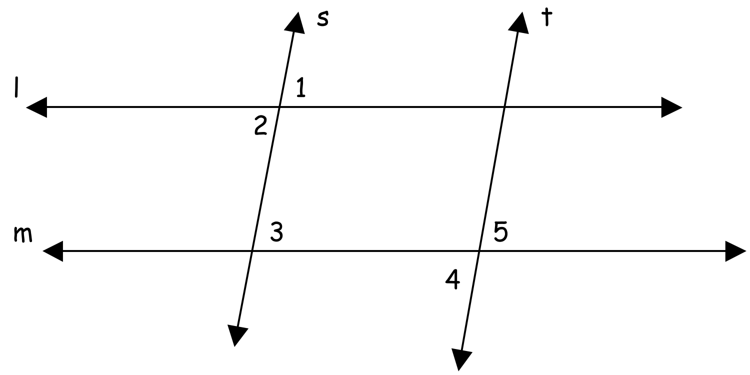


Assume a // b. Complete the chart.

ANGLES	TYPE	≅, SUPPL., OR NONE
1. ∠1 and ∠14		
2. ∠2 and ∠15		
3. ∠7 and ∠9		
4. ∠9 and ∠16		
5. ∠10 and ∠17		
6. ∠16 and ∠14		
7. ∠9 and ∠14		
8. ∠18 and ∠19		
9. ∠1 and ∠16		
10. ∠3 and ∠8		
11. ∠6 and ∠9		
12. ∠12 and ∠13		
13. ∠7 and ∠11		
14. ∠6 and ∠8		
15. ∠4 and ∠13		

Parallel line Proofs

Diagram for #1-4



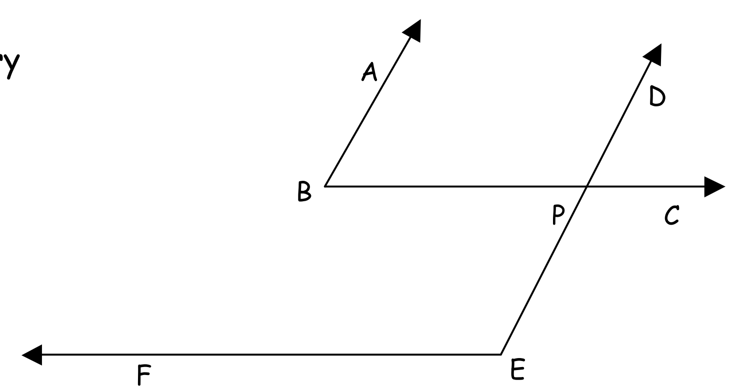
1. Given: l // m ; s // t
   Prove: $\angle 2 \cong \angle 4$

2. Given: l // m ; s // t
   Prove: $\angle 1 \cong \angle 5$

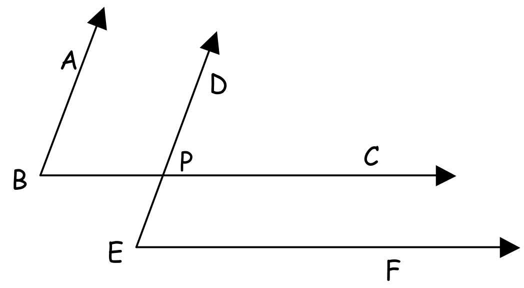
3. Given: l // m; $\angle 1 \cong \angle 4$
   Prove: s // t

4. Given: l // m; $\angle 2 \cong \angle 5$
   Prove: s // t

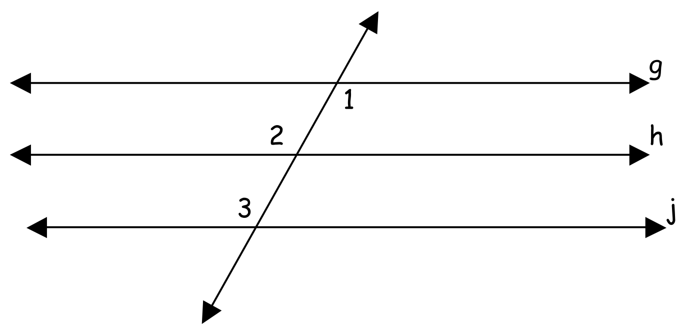
5. Given: $\overrightarrow{BC} // \overrightarrow{EF}$; $\overrightarrow{BA} // \overrightarrow{ED}$
   Prove: $\angle B$ and $\angle E$ are supplementary



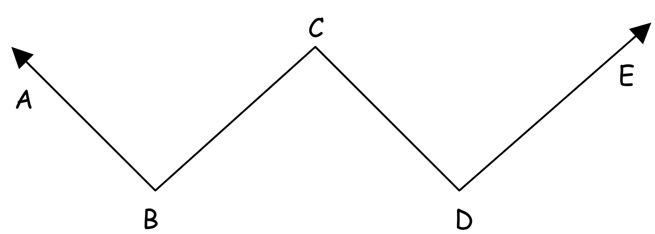
6. Given: $\overrightarrow{BC} \parallel \overrightarrow{EF}$; $\overrightarrow{BA} \parallel \overrightarrow{ED}$
Prove: $\angle B \cong \angle E$



7. Given: g // h; g // j
Prove: $\angle 2 \cong \angle 3$



8. Given: $\overline{AB} \parallel \overline{CD}$; $\overline{BC} \parallel \overline{DE}$
Prove: $\angle B \cong \angle D$



Definitions, Postulates, and Theorems

TRIANGLES:

Congruent Triangles: Triangles with corresponding sides and angles that are congruent, giving them the same size and shape

- **CPCTC** – Corresponding parts of congruent triangles are congruent.

Five postulates used to prove triangles are congruent:

Theorems:

Scramble Proof 1

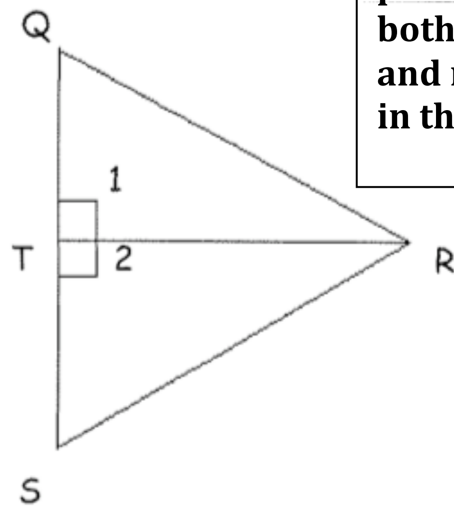
Write a correct proof by putting both the statements and reasons below in the proper order.

GIVEN:

$\overline{RT}$ bisects <QRS

$\overline{RT} \perp \overline{QS}$

PROVE: $\overline{QR} \cong \overline{SR}$



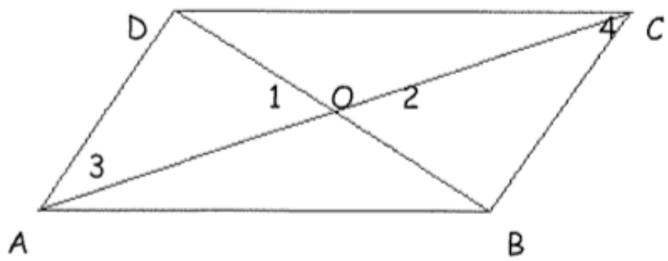
<1 ≅ <2	ASA
ΔQTR ≅ ΔSTR	CPCTC
$\overline{RT} \cong \overline{RT}$	Given
$\overline{RT}$ bisects <QRS	If 2 lines are perpendicular, then they form four right angles.
<1 & <2 are right angles	If two angles are right angles, then they are congruent.
<4 ≅ <3	Given
$\overline{RT} \perp \overline{QS}$	If an angle is bisected, then it is divided into 2 congruent angles.
$\overline{QR} \cong \overline{SR}$	Reflexive

Scramble Proof 2

Write a correct proof by putting both the statements and reasons below in the proper order.

GIVEN: $\overline{AC}$ & $\overline{BD}$ bisect each other

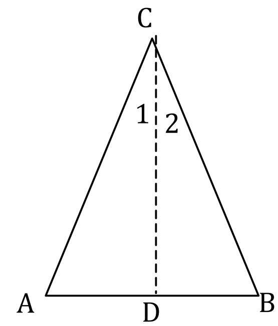
PROVE: <3 ≅ <4



ΔAOD ≅ ΔCOB	CPCTC
<1 ≅ <2	SAS
<3 ≅ <4	Vertical angles are congruent
$\overline{AC}$ & $\overline{BD}$ bisect each other	Given
$\overline{AO} \cong \overline{CO}$ $\overline{BO} \cong \overline{DO}$	If a segment bisects another segment, it passes through its midpoint.
O is the midpoint of $\overline{AC}$ & $\overline{BD}$	If a point is a midpoint, then it divides the segment into two congruent segments.

Triangle Proofs Practice

If two sides of a triangle are congruent, the angles opposite those sides are congruent. (Isosceles Triangle). Prove this theorem.



1. Given: $\overline{AC} \cong \overline{BC}$

   Prove: $\angle CAD \cong \angle CBD$

Statements **Reasons**

1. $\overline{AC} \cong \overline{BC}$

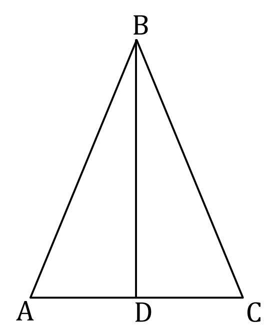
2. Draw $\overline{CD}$ bisecting $\angle ACB$

3. $\angle 1 \cong \angle 2$

4. $\overline{CD} \cong \overline{CD}$

5. $\Delta ACD \cong \Delta BCD$

6. $\angle CAD \cong \angle CBD$



2. Given: $\overline{BD}$ bisects $\overline{AC}$.
   $\overline{BD}$ is perpendicular to $\overline{AC}$.
   Prove**:** Δ ABC is isosceles

Statements **Reasons**

1. $\overline{BD}$ bisects $\overline{AC}$

2. $\overline{BD}$ is perpendicular to $\overline{AC}$

3. $\overline{AD} \cong \overline{CD}$

4. $\overline{BD} \cong \overline{BD}$

5. $\angle ADB$ and $\angle BDC$ are right angles

6. $\angle ADB \cong \angle BDC$

7. $\Delta ABD \cong \Delta CBD$

8. $\overline{AB} \cong \overline{CB}$

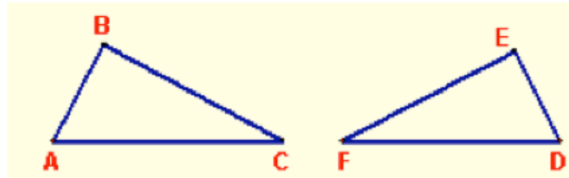
9. Δ ABC is isosceles

3. Given: $\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\angle B \cong \angle E$

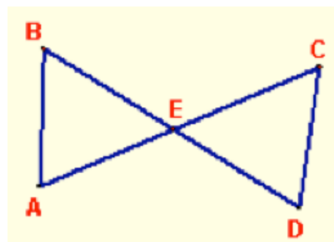
Prove: $\Delta ABC \cong \Delta DEF$



4. Given: E is the midpoint of $\overline{BD}$

$\overline{AE} \cong \overline{EC}$

Prove: $\Delta AEB \cong \Delta CED$

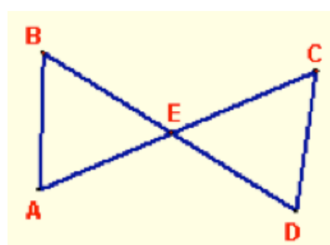


5. Given: $\overline{BA} \cong \overline{DC}$

$\overline{BD}$ bisects $\overline{AC}$

$\overline{AC}$ bisects $\overline{BD}$

Prove: $\Delta ABE \cong \Delta CDE$

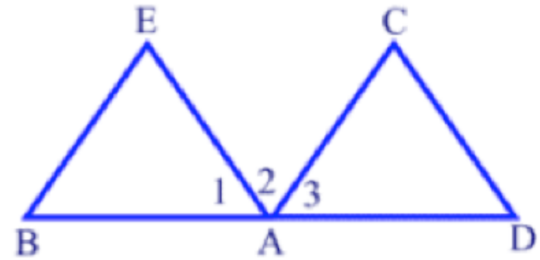


6. Given: $\angle BAC \cong \angle DAE$

$\overline{AE} \cong \overline{AC}$

A is the midpoint of $\overline{BD}$

Prove: $\Delta BEA \cong \Delta DCA$

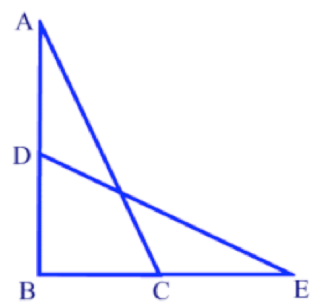


Statements **Reasons**

7. Given: $\angle A \cong \angle E$

$\overline{AB} \cong \overline{BE}$

Prove: $\overline{AD} \cong \overline{EC}$



Statements **Reasons**

Definitions, Postulates, and Theorems

QUADRILATERALS:

Quadrilateral: A 4-sided polygon.

Parallelogram: A quadrilateral with both pairs of opposite sides parallel.

Rectangle: A parallelogram with 4 right angles.

Theorems:

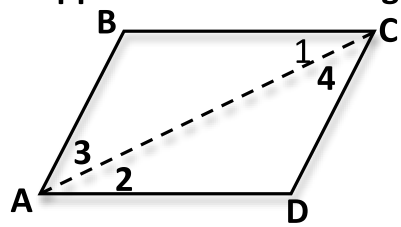
Prove Quadrilateral Theorems

If a quadrilateral is a parallelogram, the opposite sides are congruent.
Prove this theorem.

1. Given: ABCD is a parallelogram

   Prove: $\overline{AB} \cong \overline{CD}$

   $\overline{BC} \cong \overline{AD}$

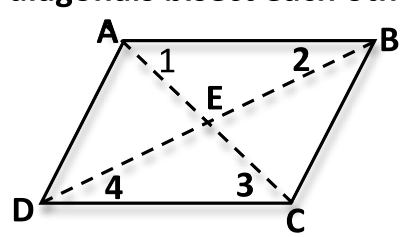


Statements	**Reasons**
1. ABCD is a parallelogram	
2. Draw segment from A to C	
3. $\overline{AD} // \overline{BC}$	
4. $\angle 1 \cong \angle 2, \ \angle 3 \cong \angle 4$	
5. $\overline{AC} \cong \overline{AC}$	
6. $\Delta ABC \cong \Delta CDA$	
7. $\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{AD}$	

If a quadrilateral is a parallelogram, the diagonals bisect each other.
Prove this theorem.

2. Given: ABCD is a parallelogram

   Prove: AE=EC, BE=DB



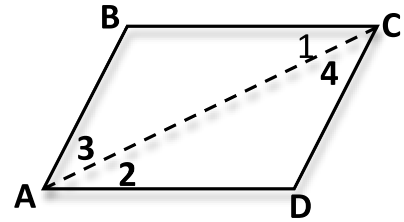
Statement	**Reason**
1. ABCD is a parallelogram	
2. AB // CD	
3. $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$	
4. $\overline{AB} \cong \overline{CD}$	
5. $\Delta ABE \cong \Delta CDE$	
6. $\overline{AE} \cong \overline{EC}$ $\overline{BE} \cong \overline{DB}$	
7. AE=EC, BE=DB	

If a quadrilateral is a parallelogram, the opposite angles are congruent. Prove this theorem.

3. Given: ABCD is a parallelogram

Prove: $\angle ABC \cong \angle CDA$

$\angle BCD \cong \angle DAB$

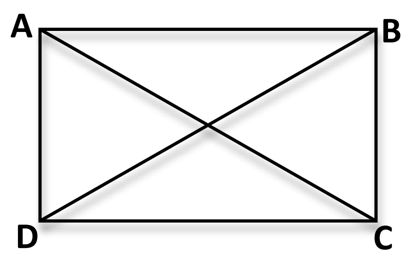


Statements **Reasons**

If a quadrilateral is a rectangle, the diagonals are congruent. Prove this theorem.

4. Given: ABCD is a rectangle

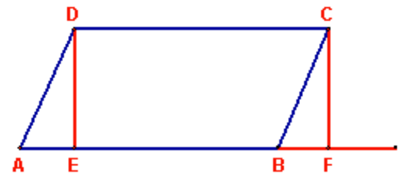
Prove: AC = BD



Statement **Reason**

Scramble Proof 3

Write a correct proof by putting both the statements and reasons below in the proper order.



Given:

ABCD is a parallelogram

$\overline{DE} \perp \overleftrightarrow{AB}, \overline{CF} \perp \overleftrightarrow{AB}$

Prove:

DEFC is a rectangle

Statements	Reasons
DEFC is a parallelogram	If two lines are perpendicular to the same line, then they are parallel
$\angle DEB$ is a right angle	Perpendicular lines meets to form right angles.
$\overline{DC} \parallel \overline{AB}$	Given
DEFC is a rectangle	If one angle of a parallelogram is a right angle, the parallelogram is a rectangle.
ABCD is a parallelogram	If both pairs of opposite sides of a quadrilateral are parallel, the quadrilateral is a parallelogram.
$\overline{DE} \perp \overline{AB}$, $\overline{CF} \perp \overline{AB}$	A parallelogram has 2 sets of opposite parallel sides.
$\overline{DE} \parallel \overline{CF}$	Given

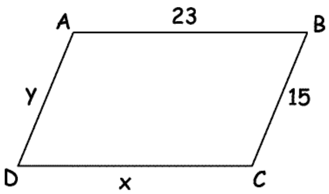
Correct Proof:

Statements	Reasons

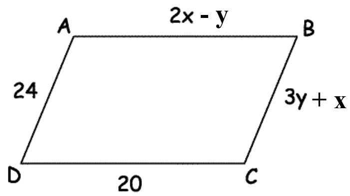
Parallelogram Practice

For all problems, assume ABCD Is a parallelogram.

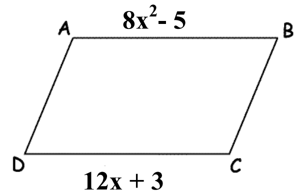
Ex 1: Solve for *x* and *y*.



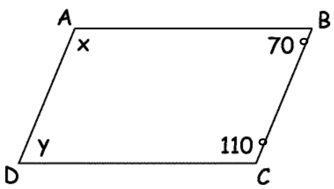
Ex 2: Solve for *x* and *y*.



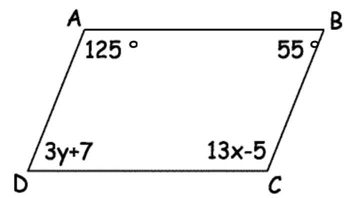
Ex 3: Solve for *x* and find AB.



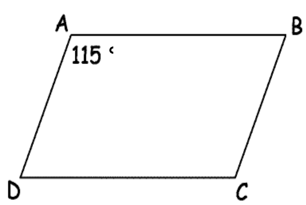
Ex 4: Solve for *x* and *y*.



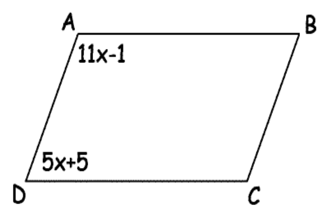
Ex 5: Solve for *x* and *y*
& find $m\angle C$ and $m\angle D$.



Ex 6: If $m\angle A = 115^\circ$, find $m\angle B$, $m\angle C$ and $m\angle D$.



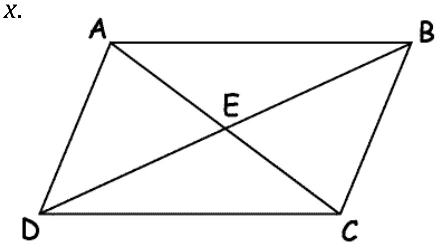
Ex 7: Find $m\angle A$ and $m\angle D$.



For examples 8 – 11, use the figure to the right.

Ex 8: If *AE* = 8, find *EC*.

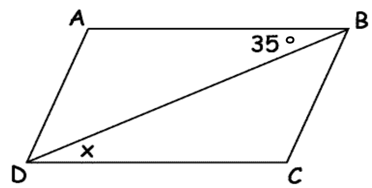
Ex 9: If *EB* = 12 and *DE* = 3*x*, solve for *x*.



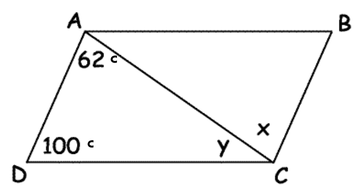
Ex 10: If *DE* = 7*x* + 2 and *EB* = 9*x* – 6, find *DB*.

Ex 11: If *EC* = 3*x* – 8 and *AC* = 4*x* +6,
solve for *x* and find *AC*.

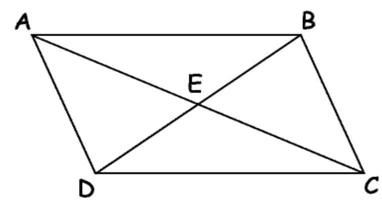
Ex 12: Solve for x.



Ex 13: Solve for x and y.



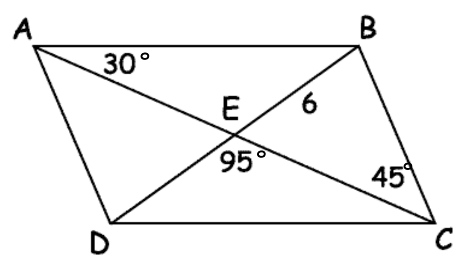
Ex 14: Given that $ABCD$ is a parallelogram, if $m\angle CDB = 20$ and $m\angle DBC = 50$, find



(a) $m\angle DBA$

(b) $m\angle DAB$

Ex 15: Given that $ABCD$ is a parallelogram, find the following:



(a) $m\angle DCA =$

(b) $m\angle DAC =$

(c) $m\angle DBC =$

(d) $m\angle ADB =$

(e) $m\angle ABD =$

(f) $m\angle CDB =$

(g) $DE =$

Introduction to Constructions

Constructions:

The word construction in geometry has a very specific meaning: the drawing of various shapes using only a pair of compasses and straightedge or ruler. No measurement of lengths or angles is allowed.

The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only compasses and straightedge or ruler. Very importantly, you are not allowed to measure angles with a protractor, or measure lengths with a ruler.

Compasses

Compasses are a drawing instrument used for drawing circles and arcs. It has two legs, one with a point and the other with a pencil or lead. You can adjust the distance between the point and the pencil and that setting will remain until you change it.

Straightedge

A straightedge is simply a guide for the pencil when drawing straight lines. In most cases you will use a ruler for this, since it is the most likely to be available, *but you must not use the markings on the ruler during constructions*. If possible, turn the ruler over so you cannot see them.

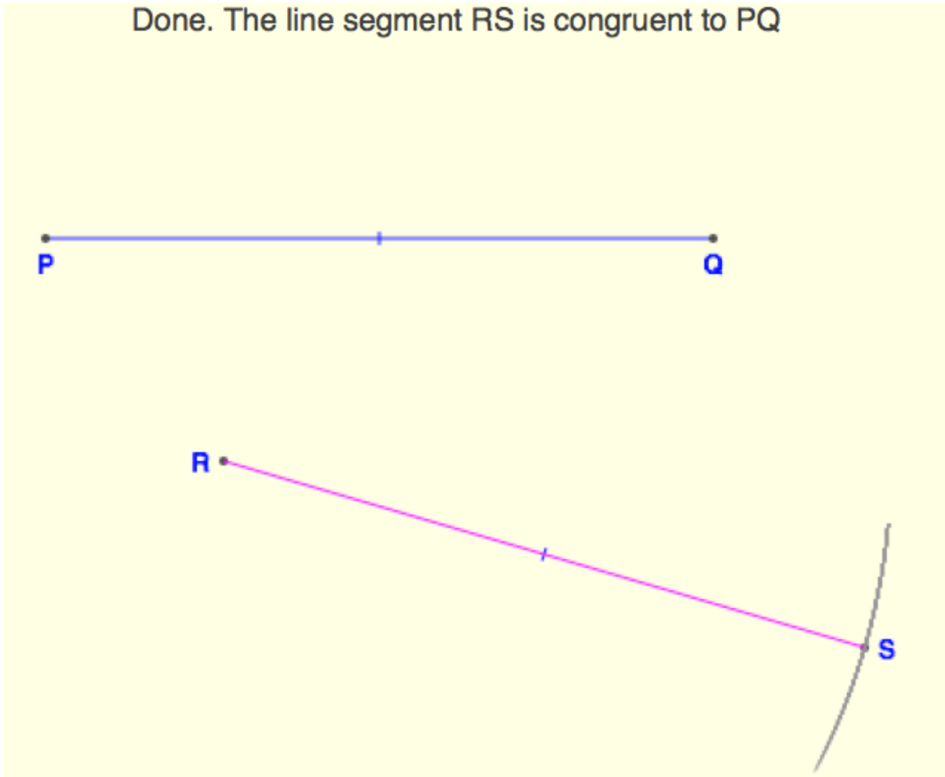
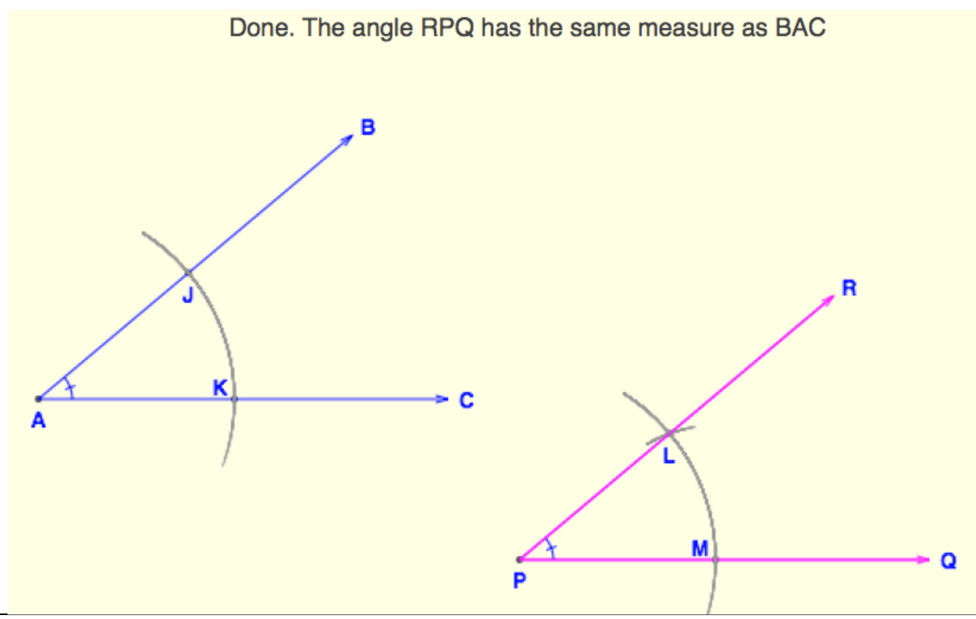
Why we learn about constructions:

The Greeks formulated much of what we think of as geometry over 2000 years ago. In particular, the mathematician Euclid documented it in his book titled "Elements", which is still regarded as an authoritative geometry reference. In that work, he uses these construction techniques extensively, and so they have become a part of the geometry field of study. They also provide insight into geometric concepts and give us tools to draw things when direct measurement is not appropriate.

Why did Euclid do it this way?

Why didn't Euclid just measure things with a ruler and calculate lengths? For example, one of the basic constructions is bisecting a line (dividing it into two equal parts). Why not just measure it with a ruler and divide by two?
One theory is that the Greeks could not easily do arithmetic. They had only whole numbers, no zero, and no negative numbers. This meant they could not for example divide 5 by 2 and get 2.5, because 2.5 is not a whole number - the only kind they had. Also, their numbers did not use a positional system like ours, with units, tens, hundreds etc, but more like the Roman numerals. In short, it was quite difficult to do useful arithmetic.

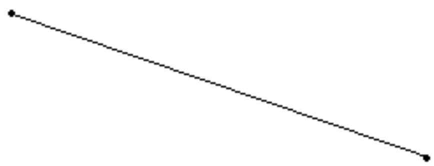
So, faced with the problem of finding the midpoint of a line, they could not do the obvious - measure it and divide by two. They had to have other ways, and this lead to the constructions using compass and straightedge or ruler. It is also why the straightedge has no markings. It is definitely not a graduated ruler, but simply a pencil guide for making straight lines. Euclid and the Greeks solved problems graphically, by drawing shapes instead of using arithmetic.

Copy a Line Segment	http://www.mathopenref.com/constcopysegment.html 
Copy an Angle	http://www.mathopenref.com/constcopyangle.html 

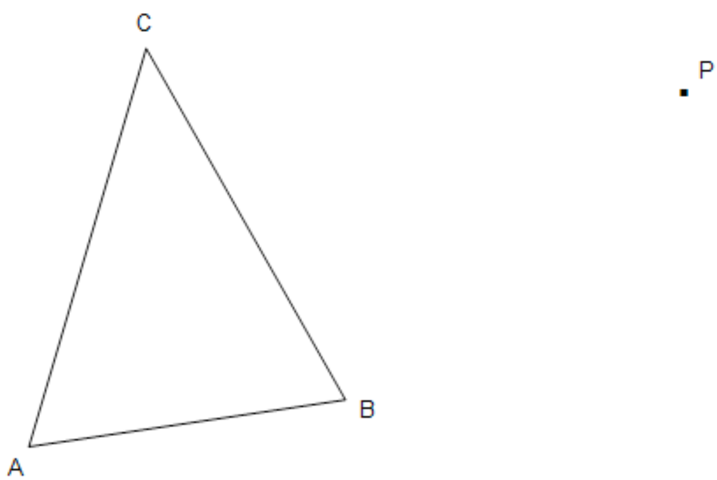
MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

Constructing a copy of a given line segment with compass and straightedge
(For assistance see www.mathopenref.com/constcopysegment.html)

1. Construct a copy of the line segment below.



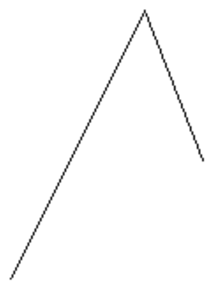
2. Construct a copy of the line segment AB from the triangle. The copy should have one endpoint at P.



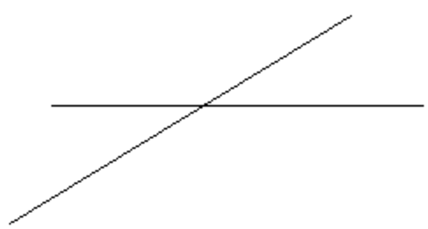
MATH OPEN REFERENCEwww.mathopenref.com NAME:

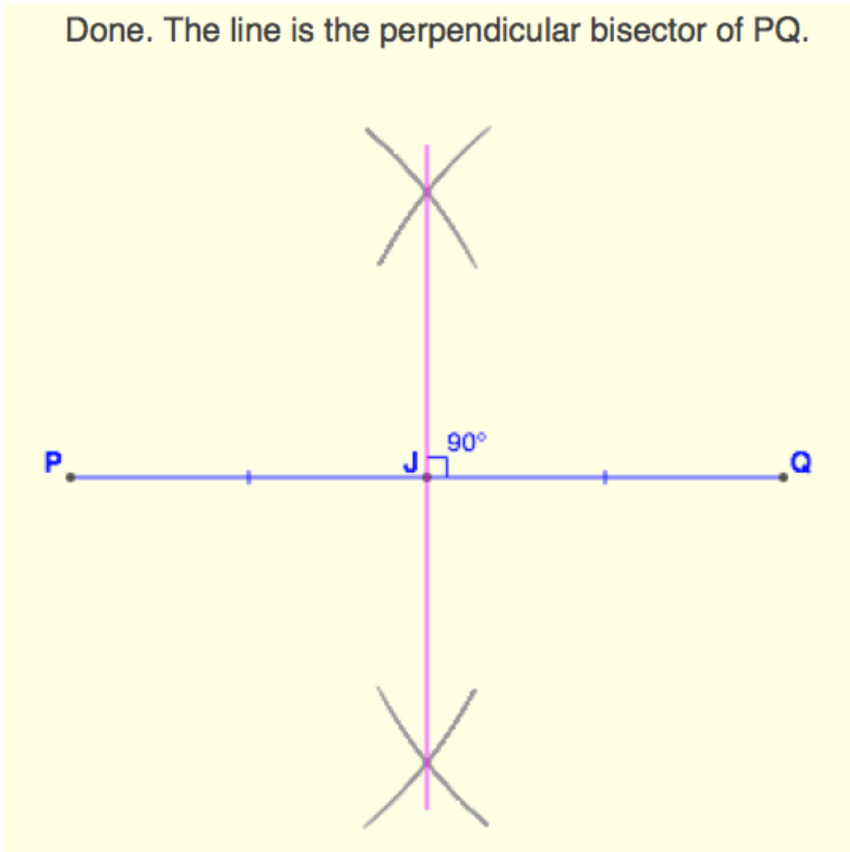
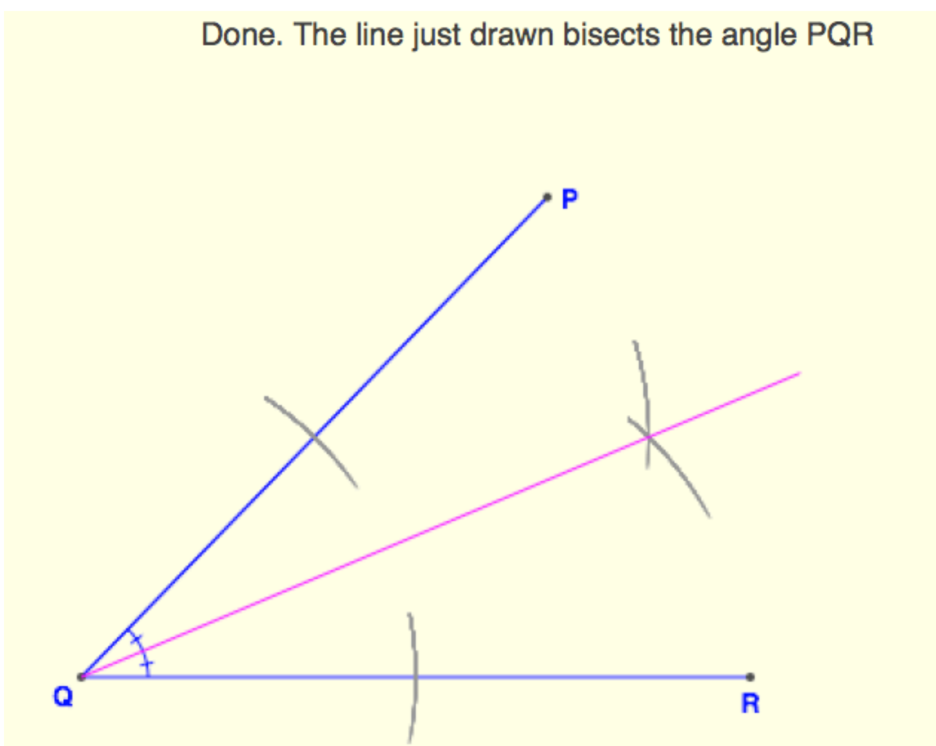
Constructing a copy of a given angle with compass and straightedge
(For assistance see www.mathopenref.com/constcopyangle.html)

1. Construct a copy of the angle below.



2. Using the fewest arcs and lines, construct a copy of the shape below. (The record: 5 arcs, 2 lines)

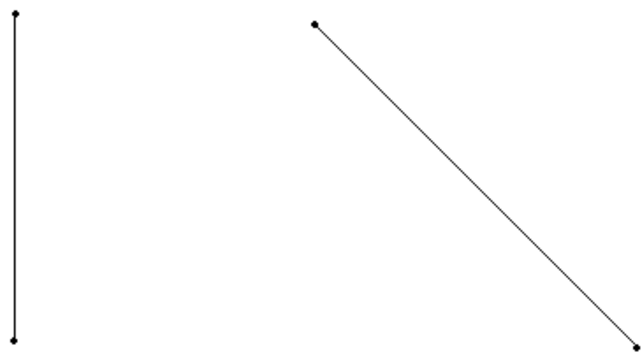


	 Done. The line is the perpendicular bisector of PQ. P J 90° Q
	 Done. The line just drawn bisects the angle PQR P Q R

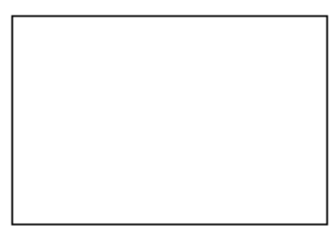
MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

Constructing the perpendicular bisector of a line segment with compass and straightedge
(For assistance see www.mathopenref.com/constbisectline.html)

1. Draw the perpendicular bisector of both of the two lines below



2. Construct the four perpendicular bisectors of the sides of the rectangle below,
   using the fewest arcs and lines. (The record: 3 arcs, 2 lines)



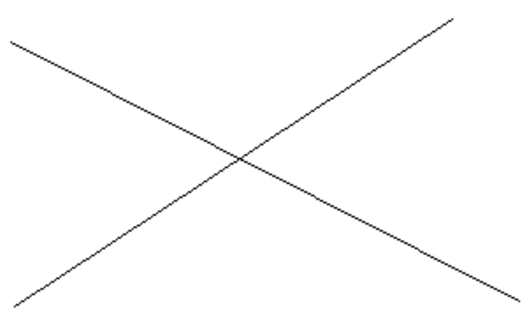
MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

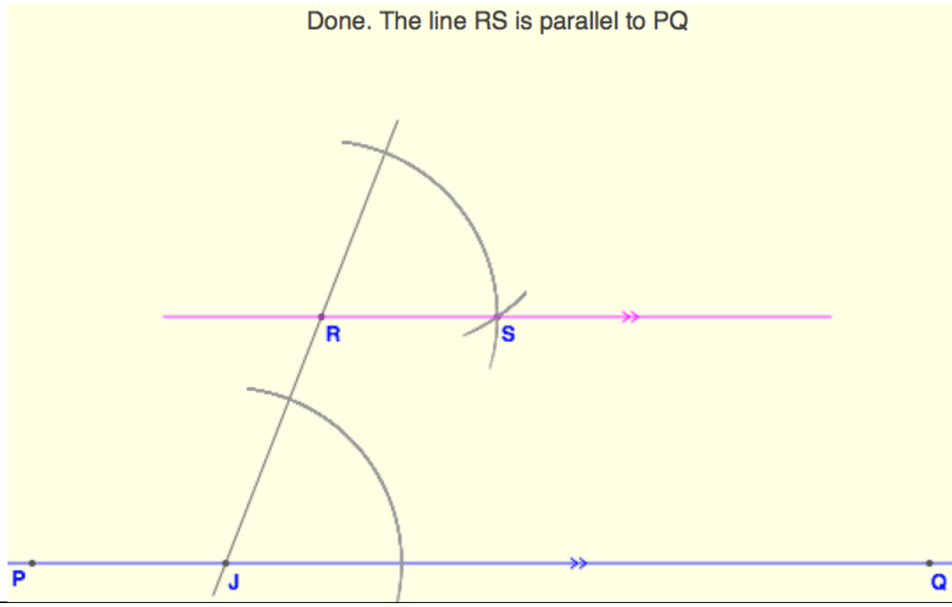
Bisecting an angle with compass and straightedge

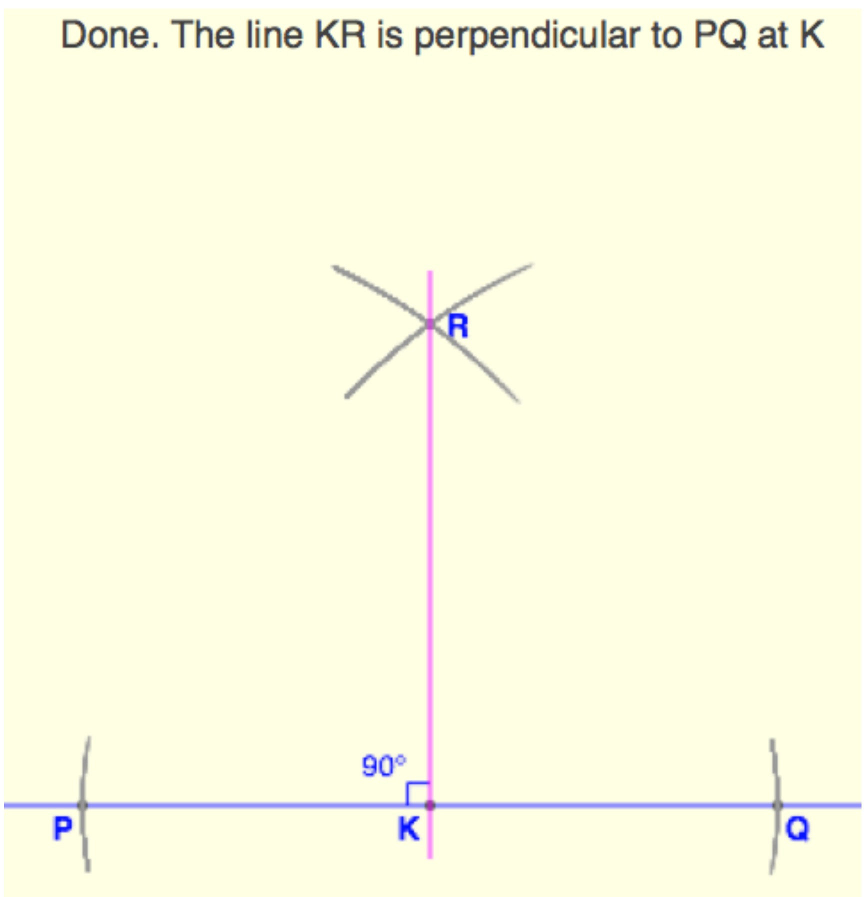
(For assistance see www.mathopenref.com/constbisectangle.html)

1. Bisect the two angles below.

2. Challenge Problem: Bisect all 4 angles, using the fewest arcs and lines. (record: 4 arcs 2 lines)



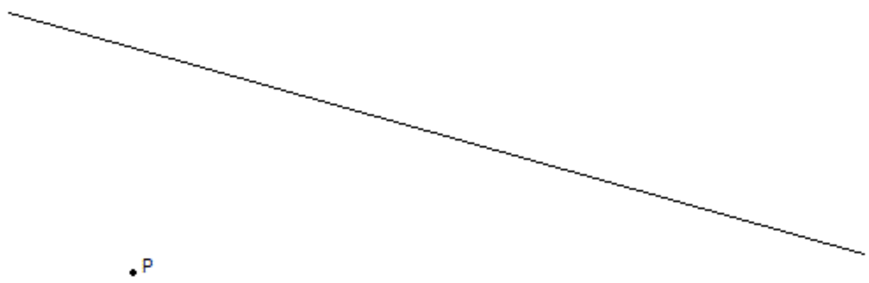

Done. The line RS is parallel to PQ
R
S
P
J
Q


Done. The line KR is perpendicular to PQ at K
R
90°
P
K
Q

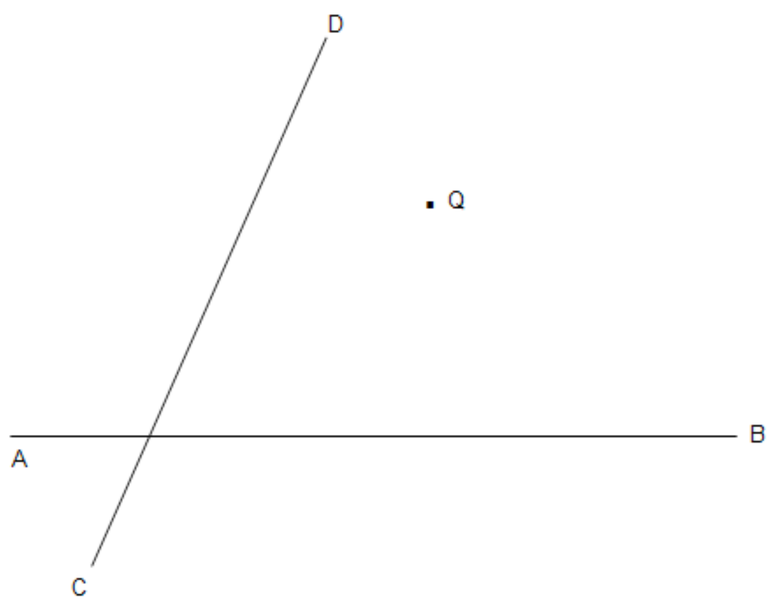
MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

Construct a line parallel to a given line through a given point with compass and straightedge
(For assistance see www.mathopenref.com/constparallel.html)

1. Construct a line parallel to the one below that passes through the point P



2. (a) Construct a line parallel to AB through Q, and another line parallel to CD also through Q
   (b) What is the name of the resulting 4-sided shape?

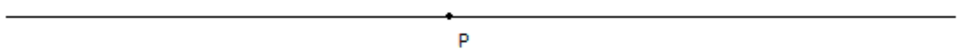


MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

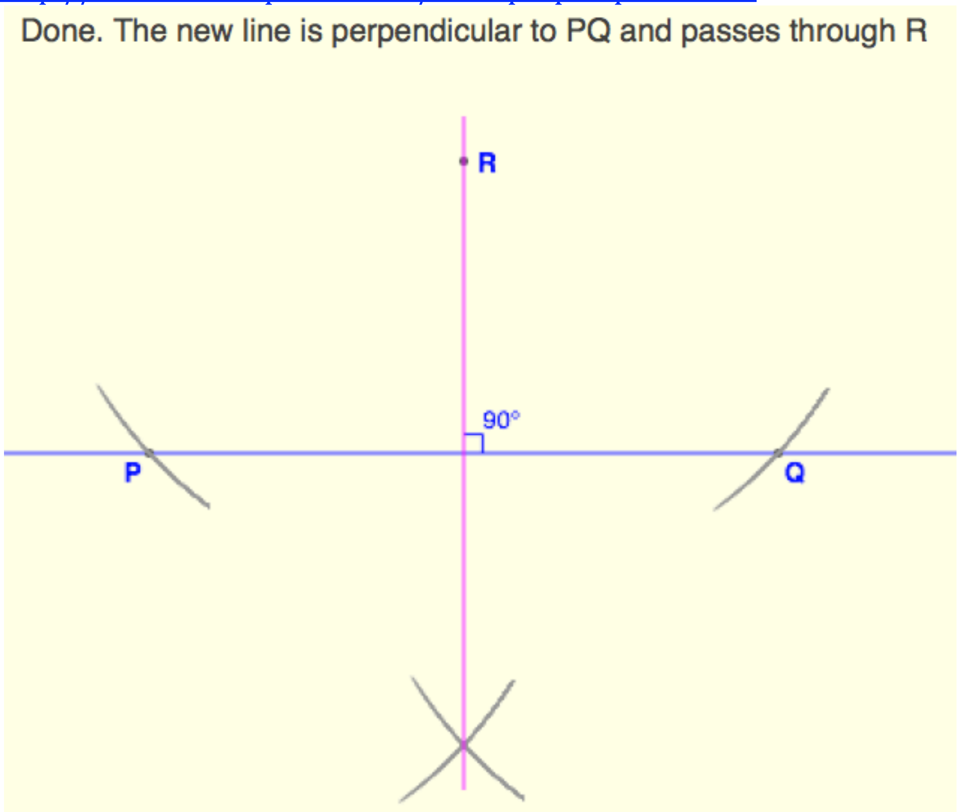
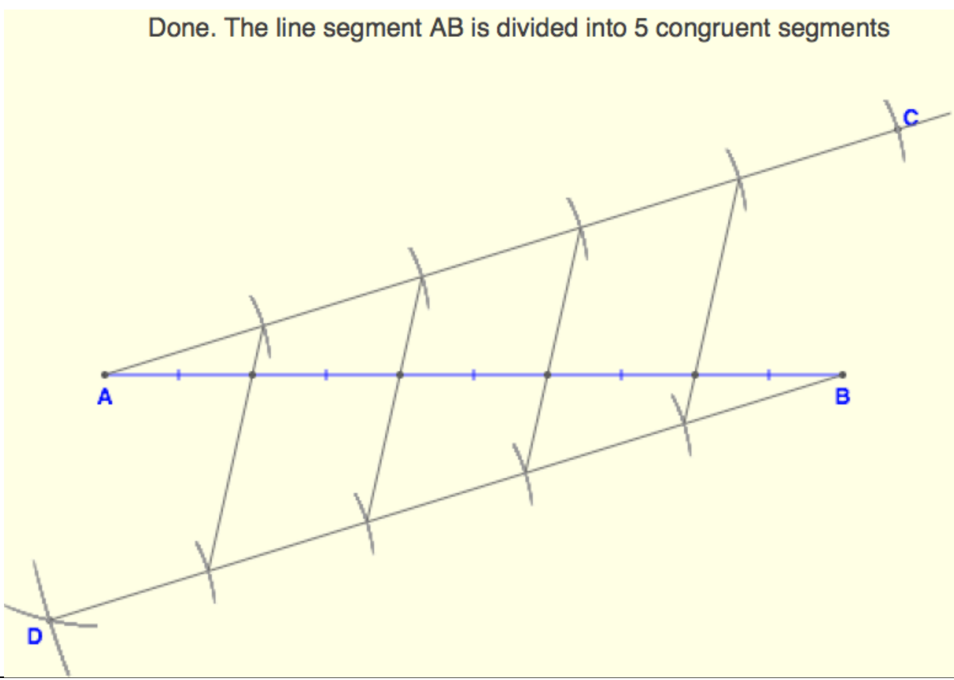
Construct a line perpendicular to a given line from a point on the line, with compass and straightedge

(For assistance see www.mathopenref.com/constperplinepoint.html)

1. Construct a line perpendicular to the ones below that passes through the point P



Math Open Reference   http://www.mathopenref.com/constructions.html

	http://www.mathopenref.com/constperpextpoint.html 
	http://www.mathopenref.com/constdividesegment.html 

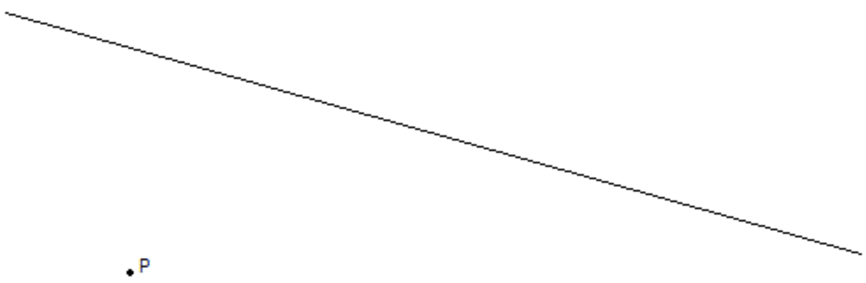
http://www.mathsisfun.com/geometry/constructions.html

MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

Construct a line perpendicular to a line that passes through a point, with compass and straightedge

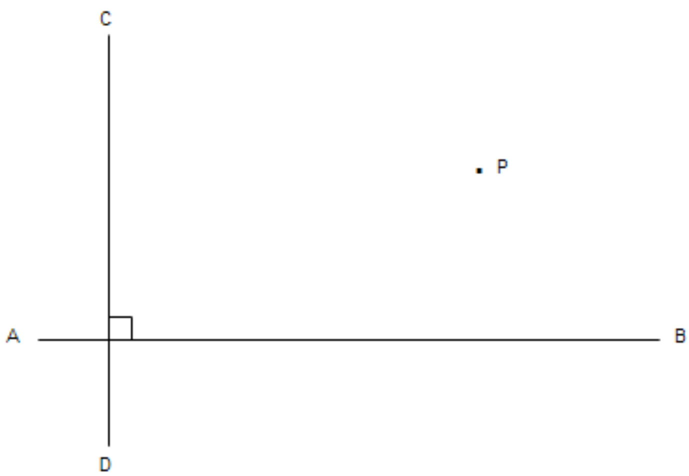
(For assistance see www.mathopenref.com/constperpextpoint.html)

1. Construct a line perpendicular to the one below that passes through the point P



2. (a) Construct a line perpendicular to AB through P, and another line perpendicular to CD also through P

   (b) What is the name of the resulting 4-sided shape? Measure its side lengths with a ruler and calculate its area.



MATH OPEN REFERENCEwww.mathopenref.com **NAME:**

Divide a given line segment into a N equal segments with compass and straightedge

(For assistance see www.mathopenref.com/constdividesegment.html)

1. Divide the line segment below into 3 equal parts

2. Divide the line segment below into 7 equal parts.